MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 102 No. 3

MEETING OF 1942 MAY 8

Professor S. Chapman, President, in the Chair

The election by the Council of the following Fellows was duly confirmed:—

Charles Edward Rhodes Bruce, M.A., B.Sc., A.M.I.E.E., 15 Savoy Street, London, W.C. 2;

Mark Howarth, Grange Mount Observatory, Newcastle, N.S.W., Australia; and

Captain Leonard F. Plugge, M.P., 5 Hamilton Place, London, W. 1.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Major Alfred Marshall Graeme-Thom, M.C. (retired), Springfield, Cox Green Lane, Maidenhead, Berks (proposed by L. J. Comrie); and

Frederick E. Lewis, B.A., B.E., Ph.D., University Lecturer in Civil Engineering, University College, Dublin, 41 St. Kevin's Park, Dublin, S. 2 (proposed by E. Wilfred Taylor).

Thirty-nine presents were announced as having been received since the last ordinary meeting, including:—

Stockholm Observatory: Annaler, Band 13, Nos. 5–11; Band 14, No. 1; Meddelande, Nos. 48 and 49 (presented by the Astronomer Royal);

Jacobo Coccæo: Epistola de Mundi quæ circumferuntur, Systematis et Novo Alio, 1670 (presented by G. Wyatt, Esq.).

The President announced the death of Dr. C. D. Perrine, formerly Director of the Córdoba Observatory, an Associate of the Society.

The President called attention to the following paragraph of a letter which had been received through the Soviet Embassy from the U.S.S.R. Society for Cultural Relations with Foreign Countries:—

"The exchange of personal letters and works between members of your Society and scientists in the Soviet Union would be welcomed."

SPECTROGRAPHIC OBSERVATIONS AT THE TOTAL SOLAR ECLIPSE OF 1940 OCTOBER 1

I. INSTRUMENT AND OBSERVATIONAL PROCEDURE

R. O. Redman

(Received 1942 March 2)

At the 1936 eclipse in Japan, despite interruption by cloud, the writer was fortunate in obtaining a useful series of photographs of the spectra of the extreme limb of the Sun and of the chromosphere. These spectrograms, taken with a fairly large slit spectrograph and of higher resolution than is commonly reached in eclipse spectroscopy, were measured and discussed by Dr. Thackeray *, and it was clear as a result of his work that further observations of this kind, made if possible with still greater resolving power, should be undertaken at the first opportunity. Soon after the 1936 eclipse the Joint Permanent Eclipse Committee commenced considering plans for the exceptionally favourable eclipse of 1940, and among the expeditions suggested was one from the Solar Physics Observatory, Cambridge, under Professor Stratton. A provisional site at Nelspoort, South Africa, was chosen, and it was intended that Dr. Thackeray should there use an enlarged version of the 1936 spectrograph to which reference has just been made. After discussion as to the best optical system for the instrument which financial and observing conditions would allow, two new spectrograph lenses were purchased from Messrs. Adam Hilger, who also refigured an additional prism selected from a number available at the Solar Physics Observatory.

After the outbreak of war, towards the end of 1939, when it had become clear that all English expeditions would have to be cancelled, the Astronomer Royal arranged that the spectrograph should be used at the eclipse by the writer, who was then in permanent residence in South Africa, and early in 1940 it was shipped to Capetown together with the necessary cœlostat and auxiliary apparatus. It had been impossible, owing to difficult conditions in England, to devote as much attention to the preparation of the instrument as for a normal eclipse expedition, so that a number of minor alterations and improvements were made after its arrival in South Africa, the work being done partly at the Royal Observatory, Capetown, and partly at the Radcliffe Observatory, Pretoria. With regard to the place at which observations were to be made, it was decided to join forces with the Royal Observatory party, which under Dr. Jackson was going to Calvinia, a small Karroo town about 250 miles north of Capetown. The camp site was on land adjoining the residence of Mr. Fred Laurence, about three-quarters of a mile north-east of the centre of Calvinia, and about five miles south of the predicted centre line of the belt of totality. A preliminary visit to the place was made in June, and the actual work of preparation there was started on August 26, some five weeks before the eclipse. Dr. C. W. Allen of the Commonwealth Solar Observatory, Canberra, had also decided to come to Calvinia to observe the corona, and was already at work on the site. The Cape instruments had been erected in June, so that other observers and helpers did not arrive at the camp until late in September.

The position of the camp, according to figures supplied by the S.A. Trigonometrical Survey Office, was 31° 28′ 01″ S.; $1^h\ 19^m\ 06^s.1$ E. The position as measured by Dr. Allen and the writer was 31° 27′ 58″ S.; $1^h\ 19^m\ 05^s.4$ E. For the eclipse prediction we used the mean value 31° 28′ 00″ S.; $1^h\ 19^m\ 05^s.7$ E., and an altitude above sea-level of 3250 feet. Following Dr. Jackson's advice, based on Cape occultation observations,

* *M.N.*, 97, 672, 1937.

the correction to the tabular longitude of the Moon was taken to be 0″·8, instead of the *Nautical Almanac* value 2″·0. With these data the computed times of second and third contacts were respectively: 1940 October 1, $14^h\ 04^m\ 42^s$ and $14^h\ 08^m\ 32^s$ U.T. Totality came over three seconds earlier than this prediction. The altitude of the Sun at mid-eclipse was 32°.

The spectrograph had three glass prisms of 3 inches aperture *, with $3\frac{1}{2}$-inch collimator and camera lenses of 8 feet focal length. The dispersion at $H\gamma$ was 3·6 A./mm. The spectrum region selected for observation was $\lambda\lambda$ 4030–5000, the choice being dictated by the fact that the definition was best and the field very nearly flat over these wave-lengths. Further to the violet some curvature of field became apparent, while absorption in the prisms rose rapidly below λ 4100. In the other direction, beyond λ 5000, dispersion and resolving power fell off very considerably, while the use of orthochromatic or panchromatic plates would have diminished the instrument's efficiency for the blue-violet.

The camera and collimator tubes were of sheet aluminium on a framework of brass angle strips, and were mounted on a frame built up of angle iron. The prisms were enclosed in a very solidly constructed teak box. The camera back was the same as for 1936 †, but the plateholders were adapted to take plates (9 by $10\frac{3}{8}$ inches) instead of films. The interior of the instrument was well diaphragmed and parts of the camera lined with black velvet, in order to cut down scattered light. The electric shutter in front of the slit was improved in design so as to be faster in operation. The times of opening or closing the shutter were recorded by a chronograph which was also registering one-second signals from an electrically maintained pendulum. Time marks from a chronometer of known error were put on the chronograph tape by hand shortly before and after totality. As in 1936, the whole mechanism of moving the plate between exposures, working the shutter and recording exposure times, was operated by turning a hand crank on the camera back.

Owing to the use of a long spectrograph camera, working at $f/32$, really good definition could be obtained on the spectrograms only if the prisms gave something near theoretically perfect performance. Hence it was essential to keep the temperature of the prisms constant to about 0°·2 C., not only during the eclipse, but also for a period at least of the order of one day beforehand. The following facts had to be considered: (i) The diurnal range of temperature in Calvinia in September and October is on the average about 14° C., and may on individual days considerably exceed even this high figure; (ii) the local prices of building materials and electric power were high; and (iii) the soil and sub-soil at the camp site were of alluvial origin, soft, easily excavated and bone dry. The easiest solution therefore was to install the spectrograph in a pit. This arrangement demanded an extra mirror in the optical train, but on the other hand there was no difficulty in placing the slit of the spectrograph in the desired orientation relative to the solar image, as there would have been at this eclipse with the collimator in the more conventional horizontal position. The pit was of dimensions about 8 by 3 by $10\frac{1}{2}$ feet deep, oriented so that the principal plane of the spectrograph lay in an azimuth 79°·3 west of north. With the collimator vertical the slit was then tangential to the solar image at the computed point of second contact. In its final position the instrument was within 1° of the orientation just mentioned, and the collimator axis within 0°·3 of the vertical. The spectrograph was wedged against the north wall of the pit by wooden struts and packing pieces hammered into position. A wooden stand supported the frame about six inches from the floor of the pit, while for extra rigidity the end of the camera was

* The prisms are over thirty years old and were once the property of Sir William Huggins. The glass is of good optical quality, but is considerably less transparent in the violet than would be expected if the prisms were of more modern manufacture. All three have been refigured within the last seven or eight years by Messrs. Adam Hilger and they now give excellent definition.

† *M.N.*, **97**, 668, 1937.

clamped securely to a heavy baulk of timber, wedged against the pit wall, like the main frame, by wooden struts hammered into place. This method of holding the spectrograph had the welcome advantage of adding much to its mechanical rigidity. The mouth of the pit was closed by a sheet of fibre board, with loose pieces to allow access to the slit or camera, and over this was a small air-space and then a roof of loose sheets of corrugated iron, anchored down with bricks and loose stones. The pit was not lined.

The slit of the spectrograph was at ground-level and east of the camera. As near as possible to the slit end of the pit was a brick pier carrying a 16-inch cœlostat. This fed a horizontal beam of light to the 12-inch Burch concave mirror of 18 feet focal length (used also in 1936) whose mounting was on another brick pier, on the other side of the pit and about 17 feet west of the cœlostat. From this the light returned to a 4-inch diagonal flat, mounted on a substantial bracket on the side of the cœlostat pier, about 3 feet above the slit. The concave was stopped down to 9 inches aperture, and the loss of definition due to using the mirror slightly off axis was calculated to be much less than that due to poor seeing. All three mirrors were aluminized. Collimation adjustments needed some care; they were made by looking from the back of the spectrograph camera, using a small telescope focused on the prisms. The last collimation check was made about half an hour before totality. The solar image was 5·1 cm. in diameter.

To keep light and dust from the interior of the instrument, a heavy black cloth bag was made to seal the gaps between the spectrograph tubes and the prism box, while a similar bag enclosed the whole camera back. It was found difficult to keep the slit and collimator lens free of dust, of which there was a great deal in Calvinia, and the slit especially had to be cleaned frequently.

The chief object of the pit was to keep the temperature of the prisms constant. In this it proved very successful. A thermograph record of the air temperature at the bottom, immediately beneath the prism box, showed that in the week preceding the eclipse the total temperature range was 14° to 16½° C., much of which was caused by opening the pit for focus tests during one cool, windy evening. The temperature range during the 24 hours preceding totality was 15°·4 to 15°·8 C. It is probable that within the thick-walled, and fairly air-tight, prism box the temperature varied less than these figures, so that the prisms themselves must have been working in almost ideal thermal conditions, a conclusion confirmed by the very good definition shown by the eclipse spectrograms.

The camera handle was worked by Dr. Zanstra, who for the purpose had to stand on a ladder half-way down the pit. Since in this position it was very difficult to hear sounds from outside, the signal for starting was given by the writer, who had charge of the cœlostat and the slit end of the instrument, and who at a prearranged time switched on an electric bell, fixed in the pit and rung once per second by the pendulum already mentioned. The bell also served to time the speed of rotation of the handle.

Since it was intended to use the spectrograms for photometric purposes, including the measurement of line profiles, the instrumental profile was determined by exposures, made with a krypton tube, a few days before the eclipse. The focus of the spectrograph was not altered between this and the time of totality, and as the temperature of the optical parts also remained unaltered, these exposures should give an accurate measure of the instrument's optical performance. Unfortunately the slit could not be left undisturbed, on account of the dust difficulty just mentioned, but the width was kept as nearly as possible the same (0·04 mm.) for the eclipse as for the krypton work.

Both the krypton photographs (exposure times 10 seconds to 30 minutes) and the eclipse photographs (exposure times 0·6 second) were calibrated by two exposures on each plate, made with a slit width 3 mm. Over the slit was a Hilger neutral tint glass wedge; the light source was cloudless zenith sky, and the exposure times 20 and 40

seconds respectively. Ilford Zenith plates, backed by the writer with a mixture recommended by Ross *, were used throughout. The eclipse exposures had to be developed at 21° C., a rather higher temperature than is generally considered advisable, but difficult to avoid in the temporary dark room used.

At the eclipse itself two sets of ten exposures each were made. These were of duration about 0·6 second and were spaced at one-second intervals. The first series commenced five seconds before the computed time of second contact. The cœlostat clock was rated to follow the Sun, but as far as possible the lunar image was kept tangential to the spectrograph slit, using the slow motion cords. During totality the image was moved to the other side of the slit, the necessary adjustment of the concave mirror being made by Professor G. G. Cillié, who was also acting as timekeeper. The second set of exposures followed, starting five seconds before computed third contact. Since the place of observation was not exactly on the centre line of the belt of totality, the cusps were only 173° apart on the Sun's disc. Hence the spectrograph slit, arranged to be tangential to the images of Sun and Moon at the computed point of second contact, was 7° in position-angle from the corresponding tangent at third contact. It may be remarked here that the task of guiding would have been simplified if the cœlostat clock had been rated to follow the Moon instead of the Sun, as had been advised by Greenwich observers, the additional trouble in initial adjustments being well worth while. An unexpected source of difficulty was the very rapid diminution of brightness in the image during the ten to twenty seconds preceding second contact. This had not been properly appreciated in 1936, owing to cloud, but at Calvinia, following the image with a low-power microscope, it was found that the eye could only partially adapt itself to the sudden change and that accuracy in setting was more difficult to achieve just at the most critical moments than at normal times.

The day of the eclipse was hot and the sky practically cloudless. There was no wind, except for a large "dust-devil" which passed over the camp about two hours before totality. The sky transparency was excellent. At first contact the definition of the solar image was bad, and although it improved a little during the partial phases, the seeing was still very unsteady for totality. It is not surprising, therefore, that the shadow bands were remarkably conspicuous.

The measured times and durations of the exposures were as follows:—

		h m s		s
First plate.	Centre of exposure	14 04 37·41	U.T. Duration	0·56
		38·40		0·61
		39·41		0·58
		40·47		0·62
		41·49		0·56
		42·47		0·58
		43·44		0·55
		44·44		0·64
		45·43		0·59
		46·31		0·48
Second plate.		14 08 27·80		0·59
		28·79		0·52
		29·69		0·55
		30·64		0·63
		31·66		0·60
		32·66		0·58
		33·67		0·59
		34·66		0·55
		35·67		0·67
		36·64		0·57

* *Ap. J.*, 74, 88, 1931.

The average duration of one exposure was $0^s.58$. The error of the times in the first column should be about $0^s.1$; relative to each other, however, their accuracy is greater than this.

As will become clear in the succeeding paper, the definition of the spectrograms was very satisfactory. Examination of the first plate showed that the image of the Moon must have overlapped the slit slightly, a fault from which the second plate is free. On the first plate Fraunhofer spectrum is visible on the first two exposures and thereafter only chromospheric spectrum, which rapidly fades from exposure to exposure. About thirty emission lines are still visible on exposure 4, but most of them are too faint to be of any value for photometric work. The second plate contains much more material. Exposure 1 has an excellent example of low chromospheric spectrum, with at least 3000 measurable emission lines, and a continuous spectrum, but no absorption lines other than self-reversals which remain above the level of the continuous. On exposure 2 we have Fraunhofer spectrum from the extreme limb. The continuous spectrum is considerably stronger than on 1, with nearly four hundred identifiable absorption lines. Emission lines are still very numerous, but the fainter of them are not easily picked out as on the first exposure. Subsequent exposures show a continuous spectrum quickly increasing in strength, with Fraunhofer lines becoming much more numerous, as well as stronger individually, and the emission lines relatively less conspicuous, although there are many still to be seen on the edges even of exposure 10. Further details with regard to the spectra will be given when necessary later.

It is perhaps optimistic under present conditions to envisage the possibilities of further work of this kind, but it is felt that, in view of the results of the following paper, someone is likely to wish to make more observations with similar, though improved, apparatus, even though the occasion has to be deferred for years. Hence a few suggestions and criticisms may not be out of place.

(*a*) The chromospheric and limb spectra are at least as complicated as the normal solar spectrum, so that the highest resolving power is essential. On the other hand, the spectrograph cannot be appreciably slower than the one described in this paper, unless photographic material becomes considerably more sensitive than at present. As a rough guide to the observer it may be mentioned that exposure times for the low chromosphere should in the blue-violet be approximately the same as for the iron arc. It is extremely desirable to cover a much wider range of spectrum than was possible at Calvinia. The λλ 3000–4000 region is particularly important.

(*b*) A very good cœlostat drive is necessary and it should be rated to follow the Moon. The slow motion control for guiding should be accurate and positive. Cords are not ideal in this respect. Any slit shutter should be designed to go behind the slit, as in front it interferes with the guiding.

(*c*) The cœlostat and concave mirror mountings should be very substantial and not easily shaken by the wind. They have to work out of doors and the observer is not in a position to choose his weather. The concave mirror mount at Calvinia had to be heavily loaded with bricks to stop vibration in even a gentle breeze.

(*d*) If it can be designed without difficulty, and if it will work without detectable vibration, a mechanical drive for the camera is recommended. Hand drive is reliable and simple, but cannot give the regularity of a machine, however skilful and careful the operator may be. Ten available exposures for each contact are barely sufficient, should the eclipse not come punctually at the predicted time.

(*e*) The moment for starting exposures may be estimated from the size of the disappearing crescent just before second contact, rather than by using the time computed beforehand.* Since the most careful prediction, with the latitude and longitude of the observing station known accurately, may be in error by some seconds, there is much to be said for this method, particularly if the number of available exposures is small.

* Dyson and Woolley, *Eclipses of the Sun and Moon*, p. 57, 1937.

This procedure, however, was not followed at Calvinia, on the ground that, since the slightest mistake endangers the whole programme, the sequence of operations for any eclipse observer should be kept as foolproof as possible. As it was, the writer found his attention quite fully occupied in the time immediately preceding totality.

(*f*) If instruments requiring thermal protection are used, a pit is strongly recommended, provided local conditions are suitable.

Acknowledgments.—The arrangement whereby eclipse equipment was made available for the use of members of the Radcliffe Observatory staff was due to the Astronomer Royal, who also supervised the packing and shipping of the instruments. Dr. Thackeray, who in normal times would have used it himself, was responsible for preparing the spectrograph at Cambridge under difficult war conditions. Of the equipment used, the spectrograph belongs to the Solar Physics Observatory, Cambridge, the cœlostat to the Royal Observatory, Greenwich, the Burch mirror to the Joint Permanent Eclipse Committee, the pendulum and chronograph to the Royal Observatory, Cape of Good Hope, the induction coil used for the krypton tube to the University of the Witwatersrand, Johannesburg, and other miscellaneous apparatus to the Radcliffe Observatory, Pretoria. Drs. Jackson and Stoy assisted with preparatory work, and Dr. Allen gave much help at Calvinia in erecting the instruments. Drs. Zanstra and Cillié aided with the actual observations. The camp site was on ground belonging to Mr. Fred Laurence, who with Mrs. Laurence showed the greatest kindness to the whole eclipse party. Their neighbours, Mr. and Mrs. van der Merwe, lent a room for photographic work, the best temporary dark room the writer has so far seen at an eclipse.

A part of the expenses was met from a grant made by the Royal Society, through the Joint Permanent Eclipse Committee, and the remainder from a grant from the Union Government of South Africa.

To all these astronomers, friends and institutions the writer wishes to express his very grateful thanks.

Summary

Successful observations of the spectra of the chromosphere and extreme limb of the Sun were made at the total solar eclipse of 1940 October 1 at Calvinia, South Africa. Details with regard to the apparatus and observational procedure are given, together with a number of suggestions and criticisms useful as a guide for future work.

SPECTROGRAPHIC OBSERVATIONS AT THE TOTAL SOLAR ECLIPSE OF 1940 OCTOBER 1

II. CONCERNING THE FLASH SPECTRUM AND ATOMIC VELOCITIES IN THE CHROMOSPHERE

R. O. Redman

(Received 1942 March 2)

The amount of observational material obtained at the 1940 eclipse as a result of the work described in the preceding paper is rather large, and there remains still a great deal not yet measured or discussed. Owing to present-day conditions and the uncertainty of being able to finish the work without delay, it has been decided to publish the results in stages, as they reach completion. The present paper deals with some matters relating to the chromospheric spectrum. It is hoped to examine the question of the extreme limb spectrum in a later publication.

Wave-lengths and Identifications.—The first task in dealing with these spectrograms was to measure wave-lengths and identify as many lines as possible. Unless cut, the plates are too large for the only measuring machine at present available at the Radcliffe Observatory, so that all wave-lengths have been obtained from measurements of microphotometer tracings at magnification 100 : 1. Details of the microphotometer, a photoelectric machine completed for the observatory by Messrs. Casella in 1939, have never been published, as it is thought that a quite sufficient number of similar instruments have already been described in astronomical and physical literature. It may, however, be mentioned here that the machine was intended to give as much positional accuracy as possible on its 100 : 1 tracing-plate ratio, and that tests showed that with this, the largest of its four magnifications, plate positions could in favourable circumstances be measured with an average error about 0·8 μ, which is equivalent to between 0·002 and 0·005 A. on our spectra. While it cannot be claimed that the procedure of measuring on microphotometer tracings is as accurate as the best work an ordinary micrometer machine can produce in experienced hands, it appears entirely adequate for our present purposes, and errors arising from the microphotometer itself are undoubtedly of less importance than those from such factors as line blends or plate grain.

The chromospheric spectrum at low levels appears to be rather more complicated than the normal Fraunhofer spectrum of the Sun's disc, so that the number of lines which can be measured and identified on a properly exposed photograph at present depends solely on the resolving power of the spectrograph. The difficulty of measurement is much enhanced by the fact that the lines vary considerably in their structure, not only from line to line, but also from height to height in the chromosphere. There are many strong Fraunhofer lines in the normal solar spectrum whose counterparts in the low chromosphere are almost unrecognizable because of intense self-reversal. This reversal, which is extremely common among the stronger lines, becomes weaker with increasing height. Blends also vary with height. The well-known line λ 4471, *He*I, is heavily blended in the lower chromosphere, and is only seen clearly and unmistakably because it persists in considerable strength to heights at which the other lines have all faded away.

Using the first exposure on each plate, but relying chiefly on plate 2, exposure 1 (referred to as 2/1), 3994 emission lines have been measured and 2968 identified. On account of the difficulties just mentioned, we cannot be sure that all the fainter lines

are real. About 1500 lines are relatively strong and give reliable measures and identifications, except where blending is unusually severe. For standard wave-lengths the M.I.T. tables * have been used, together with the Revised Rowland Catalogue, and for the rare earths the classifications given by King in his well-known series of papers in the *Astrophysical Journal*. The wave-lengths covered are λλ 4023–4928. We have ignored the question of errors in the M.I.T. wave-lengths, except in a few cases where there appear to be definite misprints.

Since there was no terrestrial spectrum on the plates, well-known and easily identified lines in the chromosphere have been used for comparison. For identification a measured wave-length had in general to be within 0·05 A. of the laboratory value. This tolerance was narrowed slightly for the shorter wave-lengths and stretched a little for a few lines of longer wave-lengths where the instrumental dispersion had become smaller. Occasionally, where the Rowland and M.I.T. wave-lengths differed by an unusually great amount, and where our measure agreed better with Rowland, the latter was taken as standard.

The following spectra are certainly present in emission: *H*I, *He*I, *Na*I, *Mg*I, *Ca*I, *Ca*II, *Sc*I, *Sc*II, *Ti*I, *Ti*II, *V*I, *V*II, *Cr*I, *Cr*II, *Mn*I, *Fe*I, *Fe*II, *Co*I, *Ni*I, *Zn*I, *Sr*II, *Y*I, *Y*II, *Zr*I, *Zr*II, *Ba*II, *La*II, *Ce*II, *Pr*II, *Nd*II, *Sm*II, *Eu*II, *Gd*II, *Dy*II, and also *CH*, *CN*. A number of the stronger unidentified lines in Rowland were also found. A further 25 atomic sources, neutral or singly ionized, have been classed as "probably" or "possibly" present. They need not be mentioned here, for the list is bound to be incomplete, owing to our restricted wave-length range. The evidence for *Na*I, *Mg*I and *Ca*II on our plates is rather weak, as none of their strong lines is in our region, but they have been included in the above list as the *D*, *b*, and H and K lines are all well-known features of the flash spectrum. A rather striking feature of our spectrograms is the abundance and strength of the rare earth lines, the overwhelming majority from singly ionized atoms.

Similar measures and identifications have been made on plate 2, exposure 2 (2/2), which shows both emission and absorption lines. It is not intended to discuss these now, but one result which concerns the chromosphere may be included here. In his well-known work on the chromospheric spectrum, Menzel † remarked on an asymmetry of the emission lines relative to the corresponding absorption, and Thackeray ‡, using the spectrograms obtained in 1936, found a systematic shift in wave-length between emission and absorption amounting to 0·021 ± 0·010 A. (24 lines), emission being of shorter wave-length.

To examine this shift, 328 emission lines and 159 absorption lines were selected on 2/2 in the region λλ 4200–4600, freedom from blends and certainty of identification being the principal criteria. The whole region was covered by four long microphotometer tracings (the instrument records on a roll of bromide paper, not sheets, so that this is possible), and the mean relative shift found was

$$\text{absorption} - \text{emission} = -0{\cdot}0007 \pm 0{\cdot}002 \text{ A.}$$

In other words, the shift was zero to within ·001 or ·002 A. The average residual for an individual emission line was 0·016 A. and for absorption 0·014 A. The bad seeing during totality was a help in this particular work, for it ensured that the centre of the spectrograph slit was uniformly illuminated with both emission and absorption spectra, so that both appear on the same tracings covering only 0·6 mm. slit length. A correction for the curvature of the slit image becomes unnecessary in these circumstances. The emission lines may be regarded as chromospheric, the absorption lines as coming from within 0″·35 of the limb ($\theta > 88°{\cdot}5$).

* *M.I.T. Wave-length Tables*, Wiley & Sons, 1939.
† *Lick Obs. Publ.*, 17, 299, 1931.
‡ *M.N.*, 97, 678, 1937.

It may be objected that what should be measured is the shift of the self-absorption within a line relative to the emission part of the same line, and not absorption lines relative to independent emission lines. This is a much more difficult task and has not been attempted. There is, however, no obvious systematic asymmetry on our microphotometer tracings, for example in the sense that the emission is always stronger on the violet side of the self-reversal than on the red. Such differences as show appear to be of a random nature and due simply to irregularities in the plate emulsion, and more especially to numerous blends. In any case, whatever may be concluded on this point, there seems no reason to believe that there is at the Sun's limb a general systematic wave-length displacement between absorption and emission lines.

Instrumental Profile.—Before considering the profiles of the chromospheric lines, it was necessary to find the instrumental profile. As explained in the preceding paper, this was determined by exposures made with a krypton discharge tube, which gives lines so narrow that their profiles measured on the plate may be regarded as entirely instrumental. The question of errors of measurement will be discussed later. Examination of Table I, where the result of these measures is to be found, shows that the total width of the instrumental profile where the intensity has fallen to half the maximum value (the half-width, abbreviated to h.w. in the rest of this paper) *, is only 0·038 mm. It is clear that the optical performance was limited more by the slit width, 0·04 mm., and by the plate grain, than by any shortcomings of the prisms or lenses. The theoretical h.w. cannot be given accurately, since the spectrograph had 3-inch prisms and $3\frac{1}{2}$-inch lenses, and the illuminated aperture was neither circular nor square, but for an infinitely narrow slit and at the centre of the plate it should be near 0·030 mm.

TABLE I

Instrumental Profile for Eclipse Spectrograph

Distance from Centre	Intensity	Distance from Centre	Intensity
0·000	1·00	0·06	0·026
·005	0·98	·07	·016
·010	·85	·08	·0107
·015	·66	·09	·0078
·020	·45	·10	·0059
·025	·30	·12	·0038
·030	·195	·14	·0024
·035	·130	·16	·0018
·040	·087	·18	·0014
·045	·065	·20	·0011
0·050	0·045	·25	·0008
		0·30	0·0005

The measured krypton lines were near the centre of the plate and it has been assumed that the instrumental profile is constant (in mm., not A.) throughout. Except for the extreme wings, which differ by something of the order of 10 per cent. from each other, the profile is symmetrical. These differences have been neglected, as they are not thought to be significant, and in any case have no appreciable effect on the present work.

Since it is often more convenient to refer to the linear width of the lines on the plate, rather than to their widths in A., the dispersion of the spectrograph at various wave-lengths is given in Table II.

* We use the h.w. extensively in what follows, as a convenient summary description of the shapes of the emission lines. Its usefulness is enhanced by the fact that the centres of these lines, contrary to the case of strong absorption lines, are the parts where our measures are most reliable, and also the parts where the effect of velocity in the line of sight shows itself most quickly. The h.w., however, is not a complete substitute for the measured profile, to which we return when necessary.

TABLE II

Spectrograph Dispersion in Relation to Wave-length

λ	A./mm.	λ	A./mm.
4100	2·71	4600	4·78
4200	3·09	4700	5·25
4300	3·48	4800	5·75
4400	3·89	4900	6·27
4500	4·32	5000	6·81

Some Narrow Chromospheric Lines.—The first line profile to be measured on 2/1 was λ 4333, *La*II, chosen because it is a fairly strong line near the centre of the plate and is at the same time very sharp. The continuous spectrum was measured and subtracted, as has been done whenever it is of sufficient strength to appear on our

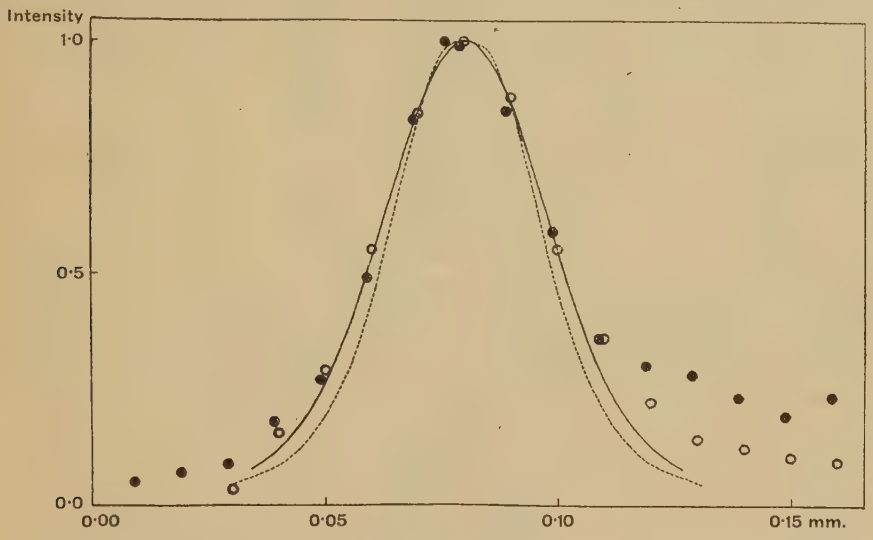


FIG. 1.—*Observations of* λ 4333, *LaII. Filled circles are from Plate* 1, *open circles from Plate* 2. *The broken curve is the instrumental profile, the continuous curve is computed for* $\bar{v} = \pm 1{\cdot}6$ *km./sec. The scale is* 1 *mm.* = 3·6 *A. The wing to the right is affected by blends with* λλ 4333·913 *PrII*, 34·030 *CH*, 34·149 *SmII.*

exposures. The h.w. of λ 4333 was then 0·043 mm., the same value being later obtained from the other plate 1/1 (fig. 1). We can come to two important conclusions immediately: (1) The spectrograph performance at the eclipse, with the moving back in action, must have been little, if at all, inferior to what it was when we photographed the krypton spectrum with the back stationary; and (2) two independent regions of the chromosphere, on opposite sides of the Sun, had very little turbulent motion, since the average velocity of the lanthanum atoms in the line of sight, parallel to the solar surface, could not have exceeded a value in the neighbourhood of ±2 km./sec. More precisely, if we assume a Gaussian distribution of velocities, the average velocity in the line of sight required to combine with the instrumental distortion to give the observed profile would be ±1·6 km./sec.

It appears unlikely, as we shall see later, that a line from the low chromosphere is ever very sharp if the corresponding line in the Fraunhofer spectrum is strong. Hence a list was made of lines in the region λλ 4200–4600, whose Rowland numbers do not exceed unity, and which in the chromosphere are reasonably strong and free from blends. They were measured on 2/1, what we use being, as before, the profile after

the continuous spectrum has been subtracted. For the results see Table III. λ 4333 has been included; it is the strongest of these lines and also the most favourable for

TABLE III

Measured Half-widths of Chromospheric Lines

λ	Source	Fraunhofer Line: Rowland No.	Fraunhofer Line: Equivalent Width (Allen)	Chromospheric h.w.	λ	Source	Fraunhofer Line: Rowland No.	Fraunhofer Line: Equivalent Width (Allen)	Chromospheric h.w.
			A.	mm.				A.	mm.
4208·983	*Zr*II	1	0·043	0·050	4434·321	*Sm*II	−1	0·013	0·050
4222·599	*Ce*II	0	·018	·043	4446·387	*Nd*II	−1	·008	·034
4256·396	*Sm*II	−1		·043	4453·706	*Ti*I	1	·044	·033
4316·801	*Ti*II	1	·042	·040	4462·985	*Nd*II	−1	·012	·036
4318·935	*Sm*II	−1	·009	·042	4467·341	*Sm*II	−1	·011	·041
4322·503	*La*II	0N	·008	·049	4470·861	*Ti*II	1	·029	·039
4333·734	*La*II	1N	·030	·043	4500·295	*Cr*I	0	·029	·055
4349·789	*Ce*II	−1	·005	·037	4560·280	*Ce*II	−1	·012	·046
4358·169	*Nd*II	0	·018	·040	4562·361	*Ce*II	0	·019	·037
4382·167	*Ce*II	−1	·012	·039	4568·312	*Ti*II	0	·019	·046
4384·294	*Sm*II	1	·045	·043	4577·174	*V*I	0	·029	·040
4398·011	*Y*II	1	·049	·044	4577·687	*Sm*II	−1		·046
4399·203	*Ce*II	−1	0·006	·042	4582·835	*Fe*II	1	·048	·049
4424·342	*Sm*II	−1		0·048	4592·058	*Cr*II	1	0·041	0·057

measurement. For all the 30 lines the average h.w. is 0·043 mm., the average wavelength 4430 A. If we confine attention to the 19 rare earth lines only, the average h.w. is 0·042 mm.; if we consider only the 22 lines whose Fraunhofer equivalent

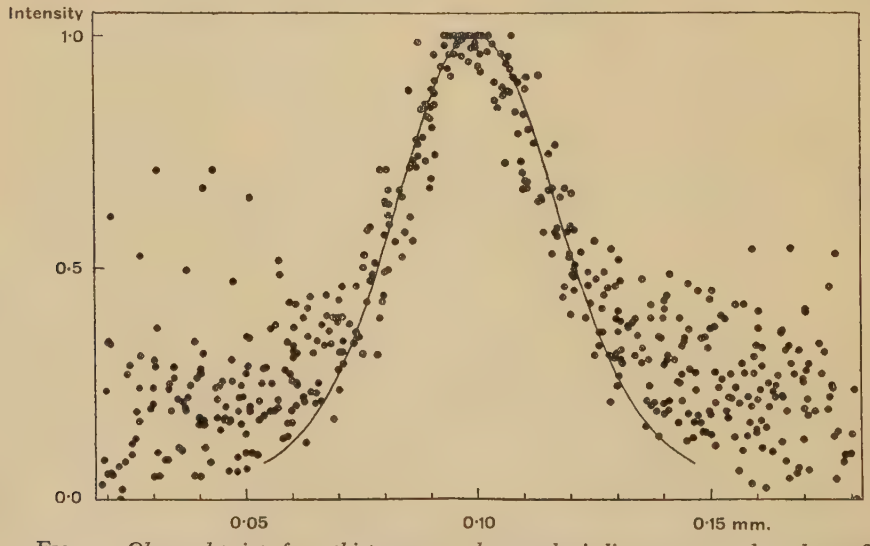


FIG. 2.—*Observed points from thirty narrow chromospheric lines, superposed on the profile computed for* $\bar{v} = \pm 1\frac{3}{4}$ *km./sec.*

widths as measured by Allen are less than 0·040 A., the average h.w. is 0·0425 mm. Restricting the choice of lines makes no essential difference to the results. It is impracticable under present conditions to publish a graph of each line separately, so that the 30 measured profiles have been reduced to a common ordinate scale, by making the maximum unity, and superposed in fig. 2. (The abscissa scale remains unaltered.)

The curve there is computed for the average wave-length λ 4430 A. by assuming an originally monochromatic radiation, modified by a Gaussian distribution of atomic velocities in the line of sight, with $\bar{v} = \pm 1{\cdot}75$ km./sec. and distorted by the spectrograph in accordance with the instrumental profile of Table I. This gives 0·043 mm. for the h.w., agreeing with the average from the 30 lines, and it can be seen that the upper part of the curve agrees very satisfactorily with observation. The growing tendency of the measured points to lie above the curve as one goes below intensity 0·5 is believed to be almost entirely due to other lines adjacent to those measured.

The Neutral Helium Line λ 4471.—The only helium line on our exposures which is favourable for measurement is λ 4471, and even this cannot be used at low levels, on account of very heavy blends, the effect of which is accentuated by a weakening of the helium line itself in the very low chromosphere. The measured h.w. at five different settings along the slit on 2/1, above the level at which blends are troublesome, were 0·077, ·088, ·089, ·096, ·096 mm., the mean being 0·091 mm. Owing to the use of a slow spectrograph the line cannot be followed to any very high chromospheric level, and the circumstances under which the spectrograms were obtained make it impossible to estimate heights except extremely roughly. As far as we can tell, these measures refer to a height about 1500 km., using the word "height" in the conventional manner. Computing the average line of sight velocity in the same manner as before we find $\bar{v} = \pm 7{\cdot}5$ km./sec. On the other plate six measures, two on 1/1, the others on 1/2, 1/3, 1/4 and 1/5, gave values for the h.w. 0·96, 1·22, 1·07, 1·05, 0·90 and 0·81 mm. respectively, with a mean value 1·00 mm. This gives $\bar{v} = \pm 8{\cdot}5$ km./sec.

For two independent reasons these helium profiles are probably a little too wide: (1) We have taken no account of the fainter component of λ 4471, apart from very slight smoothing of the curves; and (2) to avoid blends we had to measure at heights where the continuous spectrum is relatively weak, in some cases too weak to be recorded on the plate, although possibly still present in sufficient strength to affect the line profile. Inspection of the characteristic curves of the plates showed that the continuous spectrum would be almost impossible to measure when its intensity is less than one-tenth of the maximum intensity in the helium line. The effect of an unmeasured continuous spectrum is to flatten the profile. The rather large scatter between the individual values given above for the line width may in part be due to this cause.

Our measures may be compared with those Unsöld * made on the *He*I line λ 5876, the first of the series of which λ 4471 is the second member. His measures were not reduced in quite the same way as ours; he used the mean square and not the average velocity, while the 16·1 km./sec. of his paper is a space velocity, not the component in the line of sight. It is best to go back to the original measures, from which we find $\bar{v} = \pm 9{\cdot}1$ km./sec. Considering that Unsöld made no correction for instrumental distortion, while his measures were made in full daylight, and not at an eclipse, the agreement with our velocity (8 km./sec., using both plates) is not wholly unsatisfactory.

Hydrogen Lines.—When measuring the shapes of the hydrogen lines we encounter a difficulty it has been possible until now to ignore. In none of the lines considered so far is there appreciable self-reversal, although this statement is by no means true for chromospheric lines in general. The majority of strong lines, especially those corresponding to strong Fraunhofer lines, show some signs of self-reversal, which in particular cases, at low levels, may weaken and distort the line to an astonishing extent. There is a marked correlation of the strength of the reversal with the Fraunhofer intensity, and it is for this reason that for the measures described above we selected a list of lines with Rowland numbers not exceeding unity. Leaving for later consideration the question of how the self-reversal is produced, we need remark only on one fact, viz. that it has a tendency to diminish as one proceeds from lower to higher layers of the chromosphere. It is a race between a general weakening of the line and a falling off

* *Z. f. Ap.*, 3, 77, 1931.

of the reversal; sometimes the latter wins, but in certain cases, fortunately including the hydrogen lines, we can get rid of practically all the reversal before the line becomes too faint to be recorded on our spectrograms. To reach chromospheric layers as high as possible, the microphotometer slit was shortened to 0·15 mm. (as projected on the

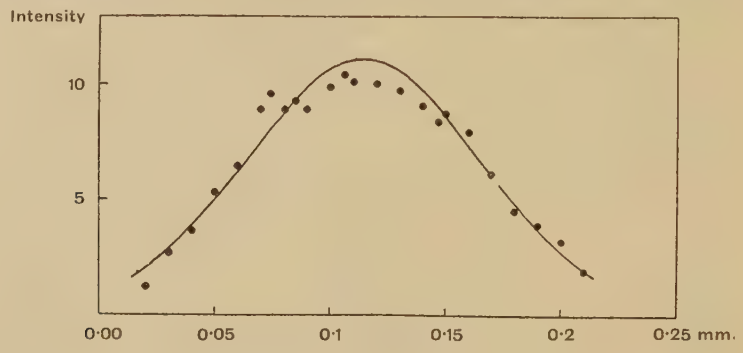


FIG. 3.—*Observed points on the profile of Hβ, with the curve computed for* $\bar{v} = \pm 14$ *km./sec.*

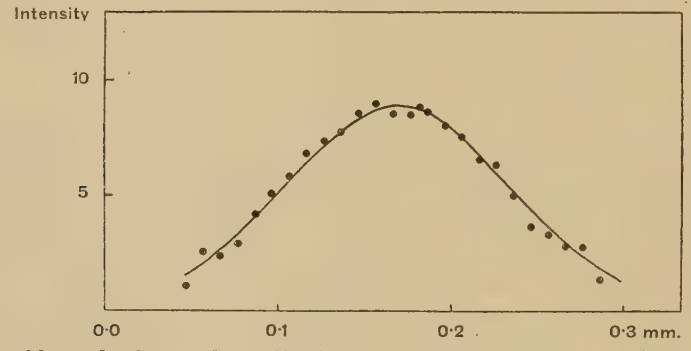


FIG. 4.—*Observed points on the profile of Hγ, with the curve computed for* $\bar{v} = \pm 12\frac{1}{2}$ *km./sec.*

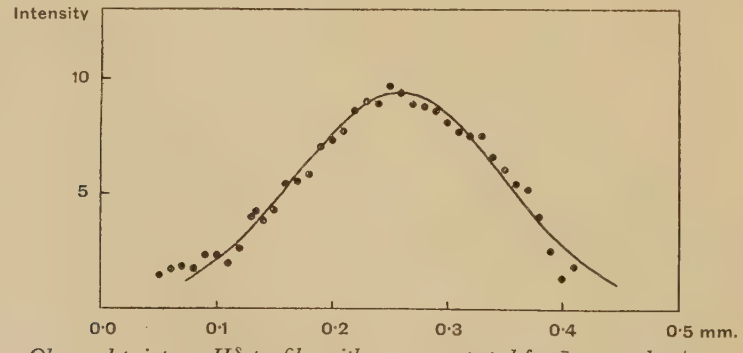


FIG. 5.—*Observed points on Hδ profile, with curve computed for* $\bar{v} = \pm 14$ *km./sec.*

plate) and widened to 0·02 mm., twice the standard setting, the lines being very broad. This arrangement minimizes the trouble that at the ends of the hydrogen lines, where we wish to measure, the intensity falls off rather rapidly, so that with a longer microphotometer slit one cannot avoid averaging over an undesirably large range of photographic density. As with helium, it is impossible to make anything but a very rough estimate of the "height," but it is again about 1500 km. The profiles from these tracings are given for *Hβ*, *Hγ* and *Hδ* respectively in figs. 3, 4 and 5. Observed points are

shown together with curves computed as before from a Gaussian distribution of line of sight velocities, plus instrumental distortion, the latter being relatively small for these lines. We find average velocities as follows:—

$H\beta$	$\bar{v} = \pm 14$ km./sec.
$H\gamma$	$12\frac{1}{2}$,,
$H\delta$	14 ,,

The fit with observation is satisfactory. There are indications of some self-reversal still remaining in the case of $H\beta$. (We expect the reversal to diminish from $H\beta$ to $H\delta$.) For the three lines together $\bar{v} = \pm 13{\cdot}5$ km./sec. It was found with $H\gamma$ and $H\delta$ that at slightly lower levels, with some self-reversal present, the outer part of the profile could still be fitted quite well to a curve computed for $\bar{v} = \pm 13\frac{1}{2}$ km./sec.

Errors of Measurement.—It is desirable at this stage to review our photometric procedure in order to obtain some idea of the reliability of the results just described. In skeleton, the method has been to measure lines on a plate calibrated with additional exposures taken through an optical wedge, and to correct the profiles so obtained by determining the instrumental distortion from krypton spectra photographed two or three days before the eclipse.

The first point to consider is our tacit assumption that the performance of the spectrograph at the eclipse was identical with that shown by the krypton photographs. As regards the temperature of the optical parts, or focus adjustments, we may be confident that there was no change which would lead to any detectable variation of the instrumental profile. On the other hand, the krypton exposures were taken with the camera mechanism stationary, the eclipse exposures with it working, and possibly setting up some small vibration in the instrument. There also may have been a slight change of slit width. As mentioned in the preceding paper, the slit had to be opened frequently for cleaning, on account of the dusty conditions at Calvinia, and unfortunately it did not have a very reliable zero reading. As much care as possible was given to keeping the width constant, but it is difficult to say to what degree of accuracy this aim was achieved. However, the width had been chosen on the basis of trials which showed that not much improvement in spectrum definition was obtained by using a narrower setting, so that the rate of change of instrumental profile with slit width was fairly small. In any case we can say that the eclipse performance was not far short of the krypton performance, since we have measured chromospheric lines with a h.w. exceeding the instrumental h.w. by only 5 μ. We can, moreover, set fairly narrow limits to the small velocities derived from these lines. In the case of λ 4333, *La*II, for example, we found $\bar{v} = \pm 1{\cdot}6$ km./sec. Now the spectrograph could never do more than give theoretical performance for an infinitely narrow slit, and for this limiting case $\bar{v} = \pm 2{\cdot}9$ km./sec. At the other limit the whole observed width of the line is instrumental, in which case $\bar{v} = 0$. (This neglects systematic photometric errors such as are considered below.) On the whole it seems more probable that the eclipse performance was slightly worse than krypton performance, than that it was better, *i.e.* $\bar{v} < 1{\cdot}6$ km./sec. is rather more likely than $\bar{v} > 1{\cdot}6$ km./sec. Before leaving this point it should be added that the instrumental profile includes distortion due to the photographic plate and to the microphotometer, as well as the spectrograph proper. Microphotometer adjustments have been kept as uniform as possible, and this contribution to the instrumental profile, as well as that of the plate, should have remained satisfactorily constant throughout.

The time-intervals between separate exposures on one plate, including calibrations, were kept small relative to the time elapsing between exposures and development, so as to minimize any effects due to latent image changes. Our plates were all brushed during development, to reduce Eberhard effect. If present, this should be of comparable amount in both eclipse and krypton exposures. Both chromospheric and instrumental

profiles would then appear too narrow, but the errors should roughly cancel each other when the krypton and chromospheric lines are of the same order of sharpness.

There is also the question of errors arising from plate calibration procedure. For the calibration exposures the wedge was illuminated direct from the zenith sky. It was, of course, screened from direct sunlight; even so, more light than was necessary to fill the prisms was entering the slit, and a harmful amount might reach the plate by reflection or scattering within the instrument. As mentioned in the preceding paper, the interior of the spectrograph is well diaphragmed, and parts of the camera tube are lined with black velvet. The lenses are cemented doublets. These favourable constructional features appear to have been sufficient to suppress the false light, for if the quantity reaching the plate were at all serious, there would be a noticeable deviation from linearity in the (log I, h) relation, to be seen most at the more lightly exposed end of the wedge image. We have been unable to find more than very slight signs of this on the eclipse plates, so that errors from this source should be negligible. The wedge "constant" was determined at Pretoria after the eclipse, using the same spectrograph. Values were found at nine wave-lengths covering $\lambda\lambda$ 4100–4800. This is the fourth wedge the writer has had occasion to calibrate at various times, and it is believed that errors of the wedge constant are here sufficiently small to be disregarded.

Owing to the conditions in which this work was undertaken, insufficient time was available for experiment with the spectrograph prior to the eclipse, in order to devise a method whereby the wedge exposures could be made of the same duration as the eclipse exposures, while retaining such an essential condition as uniformity of illumination of the wedge. Relying on the known fact that a plate's characteristic curve changes slowly with exposure time, both the eclipse exposures, of average length 0·58 second, and the krypton exposures (10 seconds to 30 minutes) were calibrated with a pair of wedge exposures of duration 20 and 40 seconds respectively. In making tests on Ilford Zenith plates since the eclipse it has been found that between exposure times 30 seconds and 30 minutes the characteristic curves are identical to a fairly high degree of accuracy. Two such curves could, for instance, be fitted together between microphotometer readings 5 and 95 per cent. with a maximum difference 0·017 in log I and an average difference about 0·007. There is thus no difficulty as far as the krypton exposures are concerned.* Between exposure times 0·5 and 30 seconds, however, our tests revealed a greater change of characteristic curve. Duplicating as nearly as possible the contrast of the eclipse plates, it was found that the slope of the central part of the 30-second curve was of the order of 10 per cent. greater than that for 0·5 second, a greater difference than had been anticipated. Unfortunately it was impossible to make these tests with plates from the same batch as those used at the eclipse, but assuming that the latter had the same properties, our chromospheric line profiles contain a small systematic error, the exact amount depending on the part of the characteristic curve which has been used. Our $\bar{v}$, then, is systematically too great. If we apply the best corrections we can, $\bar{v}$ is reduced to about $12\frac{1}{2}$ km./sec. for hydrogen, $7\frac{1}{2}$ km./sec. for helium, and 1·3 km./sec. for lanthanum. Fortunately the error is not sufficiently great to alter the general significance of our results.

Are the Velocities Thermal?—If the chromosphere were composed of gas in ordinary thermal equilibrium, the distribution of atomic velocities would be Maxwellian. In one dimension the distribution for any given element would be Gaussian, while the mean velocity would be inversely proportional to the square root of the atomic weight. Neglecting for the time being the systematic error for which evidence has just been discussed, the measured average line of sight velocities are as follows:—

---

* We have neglected the fact that the krypton tube, being run off an induction coil, was a flickering and not a steady source. This should have little effect, since the period of light variation was very short relative to the total exposure time.

Hydrogen	13·5 km./sec.
Helium	8 ,,
Lanthanum	1·6 ,,
Various other heavy elements	1·75 ,,

while the hydrogen lines fit a Gaussian velocity distribution quite well. We have seen that for two reasons the measured helium velocity is probably a little too great, while for another reason the same is probably true of the elements giving small velocities, between 1 and 2 km./sec. If the velocities *were* thermal, with the average hydrogen velocity 13·5 km./sec., other average velocities would be:

He	6·8 km./sec.
Ti, Cr, Fe	1·9 ,,
La, Ce, Nd, Sm	1·1 ,,

Everything considered, the agreement with observation is quite good, without any allowance whatever being made for turbulence. It seems clear that any chromospheric turbulence distinguishable from thermal motions, *i.e.* of a kind which affects all atoms equally, irrespective of atomic weight, cannot exceed something of the order of 1 km./sec.

The measured velocities are in the line of sight. If we assume a Maxwellian distribution given by $e^{-hm(u^2+v^2+w^2)}du\,dv\,dw$, where u, v, w are velocities in rectangular co-ordinates, m is atomic weight and h is a parameter determining the scale of the motions and hence the temperature, the average one-dimensional velocity is $\bar{v}=\frac{1}{\sqrt{(\pi hm)}}$, which is the quantity we measure. The average *space velocity* is $\frac{2}{\sqrt{(\pi hm)}}$, the mean square space velocity is $C=\sqrt{\left(\frac{3}{2hm}\right)}$ and $\frac{1}{2}mC^2=\frac{3}{2}RT$, where R is Boltzmann's constant and T is the temperature.

Thus

$$T=\frac{1}{3}\frac{mC^2}{R}=\frac{1}{2hR}=\frac{\pi m\bar{v}^2}{2R}.$$

We have $R=1{\cdot}372\times10^{-16}$, while for hydrogen $\bar{v}=1{\cdot}35\times10^{6}$ and $m=1{\cdot}66\times10^{-24}$, so that $T=35{,}000°$ K.

If we apply the systematic correction mentioned at the end of the last section, the temperature reduces to 30,000°, while the distribution of average velocity with atomic weight continues to agree as well as before with the supposition that the velocities are thermal. The temperature itself is subject to a fairly large uncertainty, owing to the fact that it varies as the square of the velocity. Hence an error of the order of 10 per cent. in line width becomes an error of the order of 20 per cent. in temperature.

It will be clear that 30,000° is simply a kinetic temperature and that our observations do not demand the existence of black-body radiation for 30,000° or any approach to thermodynamical equilibrium for such a temperature. Even so, the result at first sight may seem incredible. Evidence in its favour is mentioned below, but let us first consider various difficulties. In the first place we have ignored a large number of chromospheric lines, possibly a majority, which do not fall readily into our velocity scheme. For instance, at the greatest height at which it is measurable, λ 4227, *Ca*I, shows a rather irregular structure, with h.w. = 0·086 mm. Similarly, for λ 4215, *Sr*II, the minimum h.w. is 0·105 mm., and for λ 4077, *Sr*II, 0·133 mm. All three lines are considerably broader than the temperature suggested above will explain. On the other hand, all three are resonance lines; in the Fraunhofer spectrum they are strong, with very low residual intensities for at least two of the three; in the low chromosphere they show intense self-reversal, and the line shape remains rather irregular and flattened at the greatest heights at which we can measure. It is

legitimate to suppose that in these cases we never get rid of the self-reversal and that the lines are on that account unsuitable as a guide to atomic velocities.

Other lines behave in an intermediate manner; they are fairly wide in the low chromosphere, sometimes with clear signs of self-reversal, while although they become narrower at higher levels, the width often fails to reduce to a value which would fit our velocity scheme. Examples are shown in Table IV. With the exception

TABLE IV

Line	Fraunhofer Equivalent Width (Allen)	H.w. at Low Level	Minimum h.w. Measured
	A.		mm.
4077, *Sr*II	0·340	*	0·133
4215, *Sr*II	0·270	*	·105
4227, *Ca*I	1·230	*	·086
4246, *Sc*II	0·160	0·094	·077
4468, *Ti*II	0·141	·075	·057
4515, *Fe*II	0·103	·054	·047
4520, *Fe*II	0·080	·055	·043
4534, *Ti*II	0·150	0·090	0·058

* Heavy self-reversal.

of the calcium line, λ 4227, there is a fairly well-marked correlation between the Fraunhofer intensity and the minimum width, and for the two lines weakest in the ordinary solar spectrum the h.w. approaches the values of Table III, when measurements are made at as high a level as possible.

A hard-and-fast demonstration on this point is at present impracticable, but the writer believes that such lines as those of Table IV are all more or less seriously distorted by self-reversal and that we cannot rely on them for obtaining line of sight velocities from their profiles. It is further believed that, if we can confine our attention to lines with small Fraunhofer intensities, and therefore with self-reversal likely to be small, or if we can measure lines at fairly high chromospheric levels, agreement with a kinetic temperature about 30,000° K. will be found.

Another difficulty is that the lines may be broadened by other causes, *e.g.* by Stark effect. We can estimate the Stark broadening of the hydrogen lines, using Verwey's formulæ, as quoted by Unsöld.* At the base of the chromosphere the electron and ion concentration according to Cillié and Menzel † is 4×10^{11}, while an independent estimate by Pannekoek ‡ gives 6×10^{11}. Taking the mean value 5×10^{11}, Unsöld's fig. 65 shows that the half-width for $H\gamma$ due to Stark effect must be about 0·016 A., or 1·1 km./sec. There must be some uncertainty in this estimate, but on the other hand we have measured the hydrogen lines at some distance above the bottom of the chromosphere, which would make the computed value smaller. With other elements quantitative estimates are apparently at present impossible, but it is known that for helium the effect is less than for hydrogen, and for heavier elements much less. Present ideas with regard to the densities in the reversing layer and the chromosphere indicate that Stark effect should be much weaker in the latter than in the former, where it is known that only the hydrogen lines are affected appreciably. We are entitled to conclude that, as far as available evidence goes, the Stark effect can produce only a negligible part of the observed line widths. That being the case, and since turbulence is ruled out by the narrow rare earth lines, the only natural explanation of the observed line widths appear to be that they are due to velocities of a thermal nature.

* *Physik der Sternatmosphären*, p. 180, 1938.
† *Harvard Circular*, 410, 1935.
‡ *M.N.*, **98**, 709, 1938.

This view may become more plausible on consideration of the following points:—

(1) While our observations disprove the current theory which supposes that ordinary turbulence provides the velocities, and hence the gas pressure, to support the chromosphere *, the thermal (or pseudo-thermal) velocities we observe appear to be quite adequate for the support of hydrogen and helium, although the case of the heavier elements, particularly ionized calcium, needs further examination.

(2) A high kinetic temperature for the chromosphere may provide a clue to a long-standing puzzle, viz. why such high excitation lines as those of helium, particularly the ionized helium line λ 4686, should be observed in the chromospheric spectrum.

(3) Edlén's recent work (details of which are unfortunately still not available in this country on account of the war) calls for extremely high excitation conditions in the corona. In the photosphere, on the other hand, we have conditions corresponding more or less to a temperature of 6000° C. It would appear natural to expect that the chromosphere, lying geometrically between these two, would also be intermediate in physical properties, including atomic excitation and probably also atomic velocities. The fact that the helium line λ 4471 at first increases considerably in strength as one goes from the lowest levels of the chromosphere upwards †, reaching a maximum and then fading away gradually as still higher levels are reached, suggests strongly that there is an increase in excitation conditions from the photosphere upwards. This line is from the neutral atom, so that increase of ionization with height will work in the opposite direction and tend to reduce the intensity as one proceeds upwards. Other lines show increased intensity with height also, but on a smaller scale, and with them it is difficult to say to what extent there is a real increase, other than that caused by a decrease of self-reversal. The latter appears inadequate to explain the variation in λ 4471, particularly since the change is probably still greater than what is observed, on account of masking by blends.

The writer has had many encouraging and helpful discussions with Dr. H. Zanstra during the course of this work.

Summary

A first instalment is given of results relating to the chromospheric spectrum, from photographs obtained at the 1940 total solar eclipse.

(*a*) There is no detectable general wave-length displacement between emission and absorption lines near the Sun's limb. Comparison of 328 emission with 159 absorption lines gave a mean shift, absorption minus emission, of $-0{\cdot}0007 \pm 0{\cdot}002$ A.

(*b*) The profiles of a selected number of chromospheric lines have been measured. Making allowance for instrumental distortion, which was measured by photographing the krypton spectrum, average atomic velocities in the line of sight were found which varied between 13·5 km./sec. for hydrogen and 1·6 km./sec. for lanthanum. The distribution of velocities with respect to atomic weight, and also the shapes of the hydrogen lines, are consistent with a kinetic temperature 35,000° K. Evidence is produced to show that there is probably a small systematic error in the widths of the measured profiles. Correction for this reduces the temperature to about 30,000° K.

(*c*) At the two places observed, on opposite sides of the Sun, there was no turbulent motion (other than motion of a thermal nature) exceeding about 1 km./sec. in the line of sight.

Radcliffe Observatory,
Pretoria.

* See, for example, McCrea, *M.N.*, **89**, 483, 718, 1929.

† First noticed by Menzel (*Proc. Amer. Phil. Soc.*, **81**, 107, 1939) and quite conspicuous on our spectrograms, where the line intensity at maximum is of the order of three times the intensity at the lowest levels.

NOTE ON THE IONISATION IN THE HYDROGEN CONVECTION ZONE

Martin Schwarzschild

(Communicated by Sir Arthur Eddington)

(Received 1942 May 2)

Recently, Eddington * has shown the following: In a current descending through the hydrogen convection zone of the Sun with a speed of more than 1 km./sec., the photo-ionisation is not sufficiently effective to bring about a quasi-equilibrium degree of hydrogen ionisation. In a note added to his paper, Eddington remarks that collision ionisation might be more effective than photo-ionisation in the hydrogen convection zone, but that the calculation of the collision effects had not been possible so far because of the lack of adequate physical data.

Dr. Bethe, Dr. Fermi and Dr. Teller have kindly made available a rough estimate of the cross-section for an ionising collision of an electron with a hydrogen atom. Corresponding to this estimate, the cross-section (depending on the energy of the colliding electron) is zero for an electron energy equal to the ionisation energy of hydrogen. The cross-section increases (for small excesses of the electron energy over the ionisation energy) with the square of the energy excess, and reaches a maximum value somewhat less than the geometric cross-section of the atom (10^{-16} cm.^{2}) for an electron energy of about twice the ionisation energy. For small energy excesses the cross-section can therefore be roughly represented by:

$$C(v) = 3 \times 10^{-17} \times \frac{(E - E_0)^2}{E_0^2} \quad \text{with} \quad E = \frac{m_e}{2} v^2,$$

where v is the electron velocity and E_0 is the ionisation energy of hydrogen. Of course the coefficient 3×10^{-17} gives merely a plausible order of magnitude.

Now the effectiveness of collision ionisation in the hydrogen convection zone can be estimated. The number of ionisations per cm.^{3} in the time t is given by: $n_i = N_e N_n \overline{C(v)v}t$, where N_e is the number of electrons and N_n the number of neutral hydrogen atoms per cm.^{3}. If at the two boundaries of the convection zone the equilibrium degree of hydrogen ionisation is about ·05 and ·95 respectively, one may use as a lower limit: $N_e N_n = \frac{1}{20} N_H^2$, where N_H is the number of hydrogen atoms, neutral or ionised, per cm.^{3}. If one half of the matter is assumed to be hydrogen, one has: $N_H = \frac{\rho}{2m_H} = \frac{P}{2RT}$ (R = Boltzmann's constant), where the mean molecular weight is set equal to m_H. From the above relations one gets for the time (t_i) necessary for complete ionisation:

$$t_i = \frac{40RT}{\overline{C(v)v}P} \quad \text{with} \quad \overline{C(v)v} = 3 \times 10^{-17} \times \sqrt{\frac{2}{m_e} E_0} \times \frac{4}{\sqrt{\pi}} \frac{u+3}{u^{5/2} e^u},$$

where $u = E_0/RT$ and a Maxwellian distribution was used for the electron velocities.

In the first two lines of Table I the temperatures and pressures, prevailing at levels near the upper and lower boundaries of the hydrogen convection zone, are given both for the Sun and a red giant. The values chosen for the Sun are identical with those used by Eddington. The values for the red giant were chosen from Unsöld's † computation in such a way as to obtain equal equilibrium degrees of hydrogen ionisation at

* *M.N.*, 101, 177, 1941.

† *Physik der Sternatmosphären*, p. 146, 1938.

TABLE I

	Sun		Red Giant	
	Upper B.	Lower B.	Upper B.	Lower B.
T	9000	15000	7500	11500
P	15000	35000	400	650
u	17·5	10·5	20·9	13·7
$\overline{C(v)v}$	6×10^{-18}	$1{\cdot}5 \times 10^{-14}$	$1{\cdot}4 \times 10^{-19}$	4×10^{-16}
t_i	500	0·2	7×10^{5}	200
t_a	13		$1{\cdot}3 \times 10^{4}$	
t_i/t_a	40	1/60	50	1/60

corresponding levels in the Sun and red giant. The subsequent lines in Table I give the resulting values for u, $\overline{C(v)v}$ and t_i.

If the velocity of the convection currents is assumed to be equal to the mean thermal velocity of hydrogen in the convection zone (15 km./sec.), and if the thickness of the convection zone is taken from Unsöld's computation to be 200 km. in the Sun and 2×10^5 km. in the red giant, one can compute the time (t_a) it takes for matter to descend through the convection zone. The values obtained for t_a are given in the sixth line of Table I.

The ratio of the time t_i (necessary for complete ionisation) and the time t_a (available for the ionisation) is given in the last line of Table I. The resulting values are practically identical for the Sun and the red giant. The large value for the upper boundary might indicate that even collision ionisation is not sufficient to bring about a quasi-equilibrium degree of hydrogen ionisation in the upper part of the convection zone. But this result does not seem very certain, since the actual velocity of convection (at least for the Sun) is likely to be much smaller than assumed above. The small value of t_i/t_a for the lower boundary shows that collision ionisation is sufficiently effective to bring about a quasi-equilibrium degree of hydrogen ionisation in the lower part of the convection zone. The assumption of ionisation equilibrium, made in earlier investigations for the determination of the lower boundary of the hydrogen convection zone (leading to a small value for the heat-capacity of this zone), seems therefore justified.

Summary

The effect of collision ionisation in bringing about ionisation equilibrium in the convection zone is calculated. It is found that (unlike photo-ionisation) it operates sufficiently rapidly to produce a quasi-equilibrium degree of hydrogen ionisation in the lower part of the zone.

Rutherfurd Observatory,
Columbia University, New York City:
1942 *March* 3.

CONDITIONS IN THE HYDROGEN CONVECTION ZONE

Sir A. S. Eddington, O.M., F.R.S.

(Received 1942 May 2)

In the preceding paper Mr. Schwarzschild has given an answer, which seems to be decisive, to the question raised in the postscript to my earlier paper.* Both in the Sun and in the red giants collision ionisation restricts the break-down of ionisation equilibrium to a comparatively small part of the convection zone. My original calculation gave two main consequences:—

(*a*) A large excess of high-frequency radiation in the photosphere;
(*b*) A greatly increased width of the convection zone.

Allowing for collision ionisation, (*a*) is much reduced though it may still have some importance, and (*b*) is eliminated altogether. Admitting this, it yet remains to investigate other effects of convection which were set aside because they would have been overridden by the deviation from ionisation equilibrium if it had been genuine. This further discussion leads me to conclude that the results (*a*) and (*b*) are still valid, though the machinery is quite different. I will give first the general argument and then examine it quantitatively.

In convectively unstable conditions the adiabatic change of temperature of a moving element is less than the change of temperature of its surroundings; thus at any level the descending current tends to be cooler than the ascending current. But the temperature difference which can be set up is limited by the fact that, at given pressure, the range of temperature in which the ionisation x differs significantly from 0 or 1 is rather narrow. For example, at electron pressure 3.10^4 a temperature 14,400° gives $\frac{5}{6}$ ionisation, and a temperature 11,500° gives $\frac{1}{6}$ ionisation of hydrogen. (The difference is rather larger if the comparison is made at the same total pressure.) Thus if the descending current were to fall much more than 12 per cent. below the mean temperature, it would be practically un-ionised. Clearly the cooling tendency cannot result in a temperature so low as to eliminate the cause of the cooling, namely, the progressive increase of ionisation during the descent which uses up energy normally available for heating.

Suppose first that the temperature difference is as wide as possible. Then the downward stream has low ionisation and the upward stream high ionisation; and we have conditions partially resembling those found in my former paper. In particular the upward stream will bring large quantities of ionised hydrogen up to levels where the static calculation admits very little ionisation. The recombination of the ions will generate a large quantity of high-frequency radiation just below the photosphere, and the result (*a*) will follow. Although there is no serious physical failure of ionisation equilibrium, there is an equivalent "computational failure"—that is to say, a breakdown of the usual calculation based on the mean temperature of the level.

Turning to the other extreme, suppose next that the temperature difference is small. Then the equilibrium must be radiative, because the transport of heat by convection depends on the *difference* of thermal and ionisation energy of the outflowing and inflowing matter. The distribution in radiative equilibrium is found to be, not merely unstable, but excessively unstable. A calculation showing the excessiveness will be given later; but it is sufficiently indicated by the fact that, according to Unsöld's table for the radiative model of the Sun †, the density actually decreases inwards in a large

* *M.N.*, **101**, 177, 1941.
† *Physik der Sternatmosphären*, p. 144 (1938).

part of the convective zone. In these circumstances an element circulating between the upper and lower boundaries will gain a large amount of mechanical energy at each circuit. Assuming that the upward and downward movements occur in cells some thousands of kilometres wide (as the observations of solar granulations seem to indicate), there is not much opportunity for dissipation of energy by lateral radiation or viscosity; loss by radiation is in fact excluded by the postulate that the temperature difference is small. To employ the mechanical energy the currents must penetrate deeper into the star, thereby extending the convective zone beyond the initial region of instability. The currents themselves will modify the initial (radiative) distribution, until conditions are set up in which a circulating element gains only enough mechanical energy to overcome the small dissipation due to viscosity. This involves a less steep, and therefore more extended, ionisation gradient, and a consequent increase of the width of the zone over which it maintains convection currents.

The two extreme assumptions have led respectively to the consequences (*a*) and (*b*). The actual temperature difference will be intermediate, and will entail both consequences in a rather less extreme form. We can see in a general way how the temperature difference is controlled. When the highly unstable initial distribution is replaced by a modified distribution in which the currents are only just maintained, the temperature-pressure gradient is substantially reduced, and the outflow of radiation is reduced. This leaves most of the outflowing heat of the star to be transported through the convective zone by convection. Since convection depends on the temperature difference of the upward and downward currents, a difference must be set up sufficient to enable the heat outflow to be transported by currents of reasonable speed. On the other hand, if the temperature difference becomes too wide, the progressive change of ionisation with depth disappears in one or other of the currents, so that the motive power maintaining the circulation is cut off.

The current speed at any level cannot exceed the velocity of a particle which has fallen freely from the surface under gravity. The maximum efficiency for convection corresponds to a substantially smaller speed, since the pressure and density tend to zero as the speed approaches the limit. At 10,000° the optimum speed is probably between 4 and 10 km./sec.

I shall first give calculations which show the excessive instability of the static radiative model. As in the earlier paper, the primary data are derived from Unsöld's table for the Sun. The boundaries of the convective layer are taken to correspond to hydrogen ionisation $x_1 = 0{\cdot}94$, $x_2 = 0{\cdot}04$; these are not quite the extreme values for instability, but it seems desirable to leave the currents "room to turn round." We find

$$P_1 = 35{,}000, \quad T_1 = 15{,}000°, \quad \rho_1 = 4{\cdot}1 \,.\, 10^{-8},$$
$$P_2 = 15{,}000, \quad T_2 = \ 9{,}000°, \quad \rho_2 = 5{\cdot}2 \,.\, 10^{-8}.$$

Considering an element moving non-adiabatically (taking up the surrounding temperature as it moves), we compare the work W done by the pressure with the increase ΔE of thermal and ionisation energy in the element, in passing from the upper to the lower boundary. The result is

$$W = -1{\cdot}16 \,.\, 10^{11}, \qquad \Delta E = 4{\cdot}87 \,.\, 10^{12} \text{ ergs per gm.}$$

The calculation is straightforward, except that in calculating W I take P/ρ to vary throughout as P^{α}, choosing α so as to fit the boundary values; the formula is then

$$W = \frac{1-\alpha}{\alpha} \Delta(P/\rho).$$

ΔE consists firstly of $4{\cdot}05 \,.\, 10^{12}$ ergs per gm., representing $\frac{9}{10}$ of the ionisation energy of 0·35 gm. of hydrogen (the hydrogen abundance being taken as 35 per cent.), together with the increase of thermal energy $\frac{3}{2}\Delta(P/\rho)$.

The negative value of W (corresponding to $\rho_1 < \rho_2$) makes the comparison in this case seem fantastic; but the large excess $\Delta E - W$ indicates a high degree of instability. A more definite measure is obtained as follows. The descent of an element occupies only a few minutes; and, as a rough representation, we can suppose that its temperature is adiabatically determined until it reaches the P_1 level, where it waits to take up heat before beginning the ascent. The work W' done by the pressure in this representation is rather less than W (the temperature being lower throughout), but the difference is unimportant. The heat supplied to the element at the P_1 level is $\Delta E - W'$, or approximately ΔE. Since the mechanical efficiency of the cycle is that of an engine working between the temperatures 15,000° and 9000°, the gain of mechanical energy per cycle is about $2 \cdot 10^{12}$ ergs per gm.—enough to give the element a velocity of 20 km./sec. This velocity is, of course, never attained, the mechanical energy being diverted to increasing the extent of the convective region.

Evidently, if the static radiative distribution were by some means set up initially, it would collapse catastrophically; and to assume that the convection is confined between the initial limits of instability is rather like assuming that a bomb will bring about no redistribution of matter outside the volume it initially occupies.

A more suitable initial distribution is obtained as follows. Adopting the same upper boundary, we find, by trial and error, values of T_1, P_1 such that $W = \Delta E$, T_1 and P_1 being connected by the condition $x_1 = 0{\cdot}94$ which defines the lower boundary. The result obtained is

$$T_1 = 36{,}000^\circ, \qquad P_1 = 1{\cdot}5 \cdot 10^8 \qquad (W = \Delta E = 6{\cdot}8 \cdot 10^{12}).$$

This is certainly an extreme limit to the extent of the convection zone; and it is therefore desirable to adopt a somewhat smaller depth. Judging, as well as I can, the effect of the modifications that will be found necessary, I adopt

$$T_1 = 27{,}000^\circ, \qquad P_1 = 1{\cdot}7 \cdot 10^7 \qquad (W = \tfrac{2}{3}\Delta E; \quad \Delta E = 6{\cdot}1 \cdot 10^{12}).$$

This value of P_1 may be compared with the value $6{\cdot}6 \cdot 10^5$ obtained in my discussion of the retardation of phase in Cepheid variation.* The latter value is expected to be smaller, because it refers to a star ("star A") in which gravity is less than in the Sun and because it corresponds to $x_1 = \frac{5}{6}$ instead of 0·94. The value $1{\cdot}7 \cdot 10^7$ gives a good margin for these deductions. Thus it appears that the amount of hydrogen in the convection zone is quite adequate to account for the phase retardation in the way I suggested. (The amount calculated according to Unsöld's radiative model was much too small for this purpose.)

Since the new initial distribution gives a much reduced temperature-pressure gradient, most of the outflowing heat of the Sun must be transported through the layer by convection; and this involves the setting up of a temperature difference between the two currents, as already explained. We have to enquire whether the transport can be provided by currents with reasonable speeds. Both ionisation energy and thermal energy are transported, but the latter is relatively insignificant. For example, if one stream is fully ionised and the other un-ionised, the thermal transport corresponds to a temperature difference of, say, 4000°; but it is supplemented by a transport of ionisation energy equivalent to a temperature difference of 105,000°, as well as the additional energy of the freed electrons in the upward stream.

In the Sun the heat outflow, $H = 6{\cdot}2 \cdot 10^{10}$ ergs per cm.^{2} per sec., is equal to the ionisation energy of $5{\cdot}8 \cdot 10^{21}$ hydrogen atoms per cm.^{2} of the half-area occupied by the upward currents. Taking the current speed to be ± 4 km./sec., this would involve $1{\cdot}4 \cdot 10^{16}$ ionised hydrogen atoms per cm.^{3} We may therefore adopt $N_0 = 1{\cdot}4 \cdot 10^{16}$ atoms per cm.^{3} as the minimum hydrogen density that could provide the convective transport required. In the new initial distribution the density ranges from $\rho_1 = 9{\cdot}1 \cdot 10^{-6}$

* *M.N.*, 101, 191, 1941.

to $\rho_2 = 5{\cdot}2 . 10^{-8}$. Taking half the mass to be hydrogen, the corresponding numbers of hydrogen atoms per cm.^{3} are

$$N_1 = 2{\cdot}8 . 10^{18}, \qquad N_2 = 1{\cdot}6 . 10^{16}.$$

In most of the layer there is no difficulty; but near the top the ratio N_2/N_0 evidently does not allow a sufficient margin. There is no real conflict. Proceeding downwards from the upper boundary, there is an "end region" where the temperature difference between the two streams is gradually being set up; and here the temperature gradient in the upward stream is greater than in the downward stream. Since the radiative outflow depends on dT^4/dr, or on a higher power of T if the effect of temperature on opacity is allowed for, the mean outflow by no means corresponds to the mean temperature gradient; and, although the mean gradient in the new model is less than in the radiative model, the mean outflow of radiation may well be the same. By this means the radiative equilibrium above the convective layer continues for some distance into the layer, and only gives place to convective equilibrium when a considerable temperature difference between the two streams has been set up. This end region cannot be very deep, because the radiative equilibrium requires an excess temperature gradient in the upward stream which cannot continue very far. But it is sufficient to postpone the demand for convective transport until the density has increased enough to satisfy it.

At the bottom of the end region, where the equilibrium changes from radiative to convective, the convective transport will require currents of the order 1 to 4 km. per sec. From there downwards the temperature difference, or more strictly, the ionisation difference Δx between the streams, keeps approximately steady until near the base of the convective layer. For the convective transport is proportional to $\rho v \Delta x$, and ρv should be constant in each column by the equation of continuity of fluid motion. As illustrating the conditions contemplated, we may suppose that, excluding the end region, the ionisation increases downwards from 0·0 to 0·6 in the descending column, and from 0·4 to 1·0 in the ascending column. (The assumption $W = \frac{2}{3}\Delta E$ in our adopted distribution is a rude attempt to allow for the reduction of the ionisation difference of the two levels from 1·0 to 0·6.) The velocity, amounting to say 2 km. per sec. at the top, decreases rapidly as we go downwards since it varies inversely as ρ.

In the end region Schwarzschild's results indicate that the deviation from ionisation equilibrium, treated in my earlier paper, becomes important. The consequence is that, although the temperature difference ΔT of the two streams must diminish to zero at the upper boundary, the ionisation difference $\Delta x = 0{\cdot}4$ persists throughout the end region. Thus at the 9000° level we have 5 times as much ionised hydrogen as the equilibrium calculation admits (the mean ionisation being 0·2 instead of 0·04). The recombination of all this hydrogen above the 9000° level will cause a great excess of high-frequency radiation in the outermost layers. It appears therefore that the deviation from ionisation equilibrium, although very local, has considerable importance.

The data for the Sun fit the condition for convective transport rather closely and critically. It seems to be a matter of luck that radiative equilibrium is able to continue downwards to a density just high enough to provide convective transport. It is therefore important to examine also the red giant (effective temperature 3740°) for which Unsöld has given similar data.* The corresponding results are $N_0 = 2{\cdot}6 . 10^{15}$, $N_2 = 5{\cdot}3 . 10^{14}$. It is remarkable that, notwithstanding the large change of stellar type, the critical ratio N_2/N_0 has only changed by a factor 5. We conclude that in most stars the change-over from convective to radiative equilibrium occurs smoothly as in the Sun; but it suggests itself that in the coolest giants the junction may involve some abnormality, so that there will be peculiar conditions in the outside layers.

Considered as a hypothesis, our conclusion that the equilibrium in the convective zone is convective, is by no means novel. Even so, I do not think it has hitherto been

* *Loc. cit.*, p. 146.

realised that it gives a mass of the convective layer 500 times greater than that given by the radiative model—a modification of vital importance in the theory of Cepheid variation. But there is rather more in it than that. The fact that the transport conditions in the Sun are only just satisfied shows that we have not merely been proving the obvious. The special point here brought out is that in stellar conditions convective equilibrium necessarily involves a large difference in the state of ionisation of the ascending and descending streams, which has the important consequence (*a*). Also we get a different view of the way in which the convective distribution should be calculated. The laborious step-by-step calculation of the adiabatic gradient at successive levels is seen to be an over-elaboration of one side of the problem, which neglects the much more important point that, owing to the sensitiveness of ionisation to temperature, the two columns must be treated separately in such calculations. I have introduced a simple but crude method of calculation in this paper depending on the relation of W and ΔE. This is, of course, intended only for a first approximation; but it lets us see more clearly the principles on which a higher approximation should be based.

It appears that the phase retardation in Cepheid variables has a very simple explanation. Taking the ionised state of a hydrogen atom as standard, a neutral atom has a negative ionisation energy $-E_i$; so that, if N_n is the total number of neutral hydrogen atoms in the star, we have an ionisation energy $-N_nE_i$ which is almost all above the 30,000° level. N_n is a considerable fraction, say $\frac{1}{5}$ to $\frac{1}{2}$, of the total number of hydrogen atoms N_c in the convective zone; for we have seen that the downward stream consists largely of neutral atoms—the "returned empties" in the transport of outflowing energy across the convective zone. Since the extent of the convective zone fluctuates with the pulsation, N_c will vary. The time of pulsation is long compared with the time of circulation of the currents, so that at each stage of the pulsation we can calculate N_c as though the conditions were steady. Thus N_c and N_n fluctuate, with the pulsation; and the change of energy-content of the region above the 30,000° level can be taken equal to the change of $-N_nE_i$, since the changes of thermal energy have been found to be comparatively small. To supply this variation of energy-content there must be a phase-lag between the efflux of energy at the 30,000° level and at the surface.

This seems to be the obvious explanation of the phase retardation; and it can scarcely be rejected unless there is serious reason to think that N_c is too small to account for the observed retardation. I think it is now clear that the calculation based on the radiative model, which gave too small a value, is quite unreliable. The value is certainly much greater, and the present discussion leads to an estimate which agrees satisfactorily.

Summary

The conditions in the hydrogen convection zone are reconsidered in the light of Schwarzschild's results, which show that the deviation from ionisation equilibrium is smaller than the author had supposed. It is concluded that two of the principal results of the earlier paper remain true, though they come about in a different way. These are: (*a*) a large difference in the state of ionisation of the upward and downward streams, with the result that the upward stream carries ionised hydrogen to the top of the convection zone where its recombination produces an excess of high-frequency radiation in the outer layers; (*b*) the convection zone extends much deeper into the star than the elementary calculations (Unsöld's model) had indicated. The latter result leads to a simple explanation of the phase retardation in Cepheid variables.

ON THE OPTICAL SEE-SAW DIAGRAM

C. R. Burch, B.A., Ph.D., Bristol University

(Received 1942 April 29)

1. *Introduction to the Method.*—The "optical see-saw diagram" provides a method of describing the imaging properties in the extended paraxial region of systems of centred (aspheric) surfaces of revolution. The method may prove suggestive to those who, like myself, prefer to hang their mathematical symbols on to easily visualised physical concepts. It depends on the truth in the extended paraxial region of three simple propositions, the proof of which is left to the reader. The extended paraxial region is defined as that region within which wave-front asphericities involving angular powers higher than the 4th may be neglected.

1.1. Refraction (or reflection) at a spherical surface may for any given object-distance be rendered anastigmatic and distortionless as between object and image surfaces concentric with the refracting surface, by means of a lamina of retardation proportional to the 4th power of zonal radius, placed at the centre of curvature of the refracting surface. Such a lamina, when in real as opposed to virtual object- or image-space, can often be realised by aspheric figuring on the surface of a glass plate of sufficiently small thickness: accordingly we shall refer to it, whether "real" or "virtual," as a "figured plate" or, for short, a "plate." It is not necessary to assume that the plate, before figuring, is plane, for "concaving" or "convexing" the plate is without effect on its properties in the extended paraxial region. Any actual surface not thus anastigmatised may be treated as an anastigmatised surface, plus a plate of strength equal to minus that of the missing anastigmatising plate. Thus any actual system may be treated as an anastigmat, the performance of which is altered by the presence of certain plates, some of these being the negatives of missing anastigmatising plates, and others figurings on surfaces which we have chosen to asphérise.

1.2. Any plate in object-space may be replaced by its paraxial (*i.e.* Gaussian) image in image-space, and the curvature of any plate may be neglected. The "strength" of the plate—*i.e.* the constant K in the equation: Retardation $= K$ (zonal radius)^4—is of course altered by imaging, being inversely proportional to the 4th power of the magnification with which dust, scratches, etc., on the plate, are imaged.

1.3. Let *all* the plates except those anastigmatising individual surfaces (and every surface is always to be treated as thus anastigmatised) be imaged through the system into "star-space." By "star-space" is meant any of the representative spaces of the system in which the object is at infinity—"among the stars." It is always permissible to add to the system a hypothetical perfect anastigmat of any convenient focal length, in any of the representative spaces, to project the object to infinity, and so create a "star space" into which all the plates may be imaged. (It sometimes conduces to analytical brevity to use one of the intermediate representative spaces, rather than the initial or final space, for this purpose.)

If now these plates be regarded as masses proportional to their strengths, and positive or negative in sign as the case may be, poised on a see-saw in star-space, the pivot-point being the entrance-pupil, then

(1) The spherical aberration is proportional to the total weight on the pivot-point.
(2) The coma is proportional to the unbalanced moment about the pivot-point.

(3) The astigmatism is proportional to the moment of inertia about the pivot-point.
(4) That part of the distortion which is not associable with the representation of spheres on tangent-planes is proportional to the 3rd moment of mass about the pivot-point.

For the suggestion that it should be possible to treat distortion by means of the see-saw diagram I am indebted to Dr. Linfoot.

2.1. *Application of the Method Design of Flat-fielded Anastigmat.*—To illustrate the method, let us design a flat-fielded reflecting anastigmat comprising two mirrors and one figured (pseudoplane) glass plate. (I shall refer to such systems as "Baker" anastigmats, for as I learnt, after making the analysis which I give below, James G. Baker had already given comprehensive design formulæ for systems of this type.*) We must have the radii of the two mirrors equal and opposite, so as to give zero Petzval sum: take then a concave mirror of focal length 1, radius 2, and place inside it at a distance of 0·58 a convex mirror of focal length 1. The final image of this Cassegrain combination will then focus on a plane 0·144 . . . behind the pole of the concave mirror, which is about as close as is practicable, for we envisage in a practical case a focal length of about a foot, and a concave mirror about a foot in diameter. Such a mirror would be an inch or so thick at the pole: the photographic plate would be about half an inch behind it. This gives

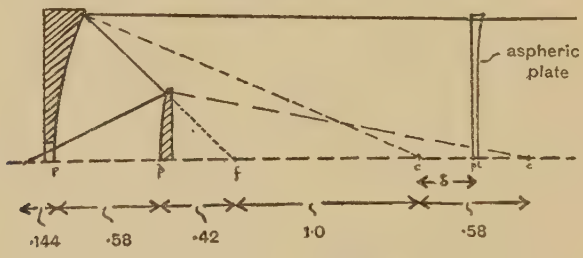


FIG. 1.—*Baker anastigmat.*

We have now to construct the see-saw diagram. Five plates are involved:

(1) The anastigmatising plate lacked by the concave mirror—taken negatively.
(2) ,, ,, ,, ,, ,, convex ,, ,, ,,
(3) The figuring on the surface of the concave mirror.
(4) ,, ,, ,, ,, convex ,,
(5) The figured glass plate which is placed in front of the two mirrors.

Plate (1) is in star-space already, at the centre of the concave mirror. Plate (2) is at the centre of the convex, *i.e.* 2·58 units in front of the concave. It images into star-space 1·63291 in front of the concave, with magnification $\frac{1\cdot63291}{2\cdot58}$. Plate (3) is in star-space already. Plate (4) is 0·58 in front of the pole of the concave. It images into star-space 1·38095 behind the pole of the concave. Plate (5) is in star-space already. Let it be distant δ in front of the centre of the concave. The strength of the anastigmatising plate lacked by a concave mirror radius of curvature ρ, object-distance u on the concave side, is $\left(\frac{u}{u-\rho}\right)^2 \cdot \frac{r^4}{4\rho^3}$ retardation at zonal radius r. Let us take as our unit of strength that lacked by our concave mirror, so that the factor $\frac{r^4}{4 \cdot 2^3}$ may be dropped.

Plate (1) then has strength -1 retardation units.

* James G. Baker, *Journal of American Philosophical Society*, **82**, 3, 339, 1940.

Plate (2) has strength $\left(\frac{0\cdot42}{1\cdot58}\right)^2$, since the convex is used at $u=\cdot42$, and it images into star-space with strength $\left(\frac{\cdot42}{1\cdot58}\right)^2\cdot\left(\frac{2\cdot58}{1\cdot6329}\right)^4=\cdot440365$ (retard). Let plate (3) have strength α (retard) and plate (4) strength β (retard) in star-space. We can now construct the see-saw diagram.

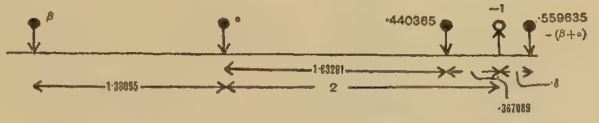


FIG. 2.—*See-saw diagram of Baker anastigmat.*

The strength of plate (5), $\cdot559635-(\beta+\alpha)$, has of course been chosen so that "$\Sigma m=0$"—*i.e.* zero spherical aberration. We may conveniently choose the centre of the concave as pivot-point. Then, setting "$\Sigma mx=0$" for aplanatism and "$\Sigma mx^2=0$" for anastigmatism,

$$3\cdot38095\beta+2\alpha+\cdot161653-\{\cdot559635-(\beta+\alpha)\}\delta=0, \qquad (1)$$

$$(3\cdot38095)^2\beta+4\alpha+\cdot0593411+\{\cdot559635-(\beta+\alpha)\}\delta^2=0. \qquad (2)$$

Eliminate δ

$$(3\cdot38095)^2\beta+4\alpha+\cdot0593411+\left\{\frac{3\cdot38095\beta+2\alpha+\cdot161653}{\cdot559635-(\beta+\alpha)}\right\}^2=0,$$

i.e.

$$9\cdot08721\beta+2\cdot82571\alpha+\cdot0593411-1\cdot86677\alpha\beta=0. \qquad (3)$$

It is possible to control the contribution to distortion (Σmx^3) and still satisfy this equation, by a suitable choice of α and β. We shall not, however, concern ourselves here with distortion, but for simplicity will consider two cases only: $\alpha=0$ and $\beta=0$.

If $\beta=0$, $\alpha=-\cdot02100$; $\delta=\cdot20607$: plate (5) strength $=\cdot538635$ retard.
If $\alpha=0$, $\beta=-\cdot00653$; $\delta=\cdot24653$: „ „ „ $=\cdot553105$ „

The alternative figurings α and β are not very heavy, and we may leave the choice to the maker. Let us calculate the asphericities involved in interference fringes. The absolute retardations involved are $\frac{\alpha r^4}{4\rho^3}$, $\frac{\beta r^4}{4\rho^3}$, r being zonal radius in star-space and ρ the radius of curvature of the primary. The depth of figurings on the mirror surfaces are half these quantities, while the amounts of "doughnut" (asphericity with respect to a sphere contacting the surface at the centre and cutting it at the edge) are $\frac{\alpha r^4}{32\rho^3}$, $\frac{\beta r^4}{32\rho^3}$. Let us set $r=6$ in., $\rho=24$ in. ($F/1$ primary; $F/1\cdot722$ final image, $F/2\cdot09$ effective, taking into account obstruction).

Then setting 100,000 interference fringes per inch, the amounts of "doughnut" involved are:

If $\beta=0$, $-6\cdot15$ fringes of doughnut (oblate figure) on the concave,
$\alpha=0$, $-1\cdot91$ „ „ „ (ellipsoidal figure) on the convex.

If we choose $\beta=0$, we can null test the concave with the aid of a fairly small spherical mirror,—the necessary ratio of radii being about 6 : 1. This is well suited to the second modification of the Twyman-Green interferometer. If we prefer to figure the convex, we had best test it with Fizeau (normal incidence) fringes against an ellipsoidal concave test-plate, which may itself be tested interfocally or by compensation methods. The

ellipsoid in question has {eccentricity}$^2 = \cdot 20986$. This alternative will usually involve building more testing gear, but is the one I would recommend if several instruments are to be built. If it is decided, in order to reduce colour error in the figured glass plate, to superpose on the negative fourth power figuring of this plate a comparable positive second power figuring—*i.e.* a spherical convexity—the anastigmatising plate lacked by this spherical convexity may not be negligible and may need to be added to the see-saw

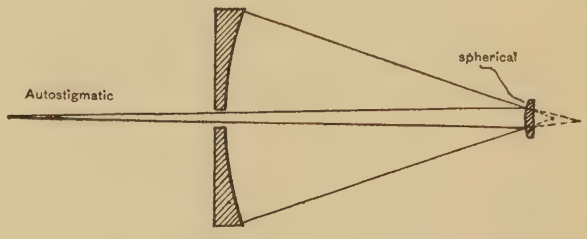


FIG. 3.—*Null test for oblate spheroid.*

diagram, so that the necessary figurings and the distance δ may be recomputed. The necessary formulæ are given in § 2.3. This completes the design problem so far as the extended paraxial region is concerned.

2.2. *Design of Approximately Flat-fielded System, with Two Spherical Mirrors and One Plate.*—Before we consider the problem of higher order aberrations, it is of interest to examine the consequence of setting $\alpha = 0$, $\beta = 0$. Equation (1) then gives $\delta = 0{\cdot}28885$ . . . and the L.H.S. of (2) (*i.e.* "Σmx^2") then becomes 0·10604. To determine the significance of this value of Σmx^2, observe that a spherical mirror of unit focal length with entrance-pupil (*i.e.* the pivot-point of the see-saw) at its pole, when used with parallel light, has $\Sigma mx^2 = -4$. Thus the astigmatism in fringes conferred on the wave-front will be $-\frac{\cdot 10604}{4}$ times that conferred by the hypothetical spherical mirror, and the astigmatic interfocal distance of the final image will be $-\frac{\cdot 10604}{4} n^2$ times that produced by the hypothetical spherical mirror, n being the magnification (1·72176) produced by the convex mirror. That is, the astigmatic interfocal distance for off-axis angle θ will be $-\frac{\cdot 10604}{4} \times 1{\cdot}72176^2\theta^2 = -\cdot 0786\theta^2$, the negative sign denoting that meridional fans focus long. Thus the field of best definition is convex towards the incident light and has depth $\cdot 0786\theta^2$ (since the Petzval sum is zero). Now, if we change the radius of curvature of the convex from 2 to 1·42, preserving distance 0·58 between the mirrors, the combination becomes monocentric and therefore (since it is spherically corrected) anastigmatic. The depth of the Petzval surface becomes $0{\cdot}2042\theta^2$, concave towards the incident light. Assuming that the astigmatism varies steadily as the radius of the convex is changed from 2 to 1·42, we observe that change of Petzval curvature contributes a fraction $\cdot 2042/(\cdot 2042 + \cdot 0786)$ of the change in field curvature.

We are therefore led to try setting the Petzval depth equal to $\cdot 0786 \times \cdot 2042/\cdot 2828\theta^2 = \cdot 0567\theta^2$. This gives for the radius of curvature of the convex, $\rho = 1{\cdot}7962$. The absolute strength of the anastigmatising plate lacked by the convex is then $\left(\frac{u}{\rho - u}\right)^2 \frac{r^4}{8\rho^3}$: as before we drop the factor $\frac{r^4}{32}$ (so that the strength unit is that of the concave anastigmatising plate). The strength of the plate lacked by the convex is then

$$\left(\frac{\cdot 42}{1{\cdot}7962 - \cdot 42}\right)^2 \left(\frac{2}{1{\cdot}796}\right)^2 = \cdot 1286,$$

and it images into star-space 1·7267 in front of the concave, with strength

$$\left(\frac{2\cdot376}{1\cdot7267}\right)^4 \times \cdot1286 = \cdot4612,$$

so that the see-saw diagram becomes

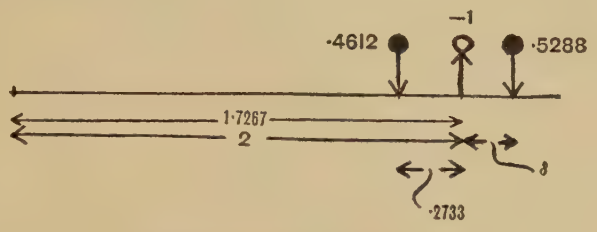


FIG. 4.—*See-saw diagram of system with both mirrors spherical.*

from which $\delta = \cdot4612 \times \cdot2733/\cdot5388 = \cdot2332$ and $\Sigma mx^2 = \cdot064$. The overall focal length is now 2·138, so that the astigmatic interfocal distance is

$$-\frac{\cdot064 \times 2\cdot138^2}{4}\theta^2 = -\cdot0731\theta^2.$$

Since the depth of the Petzval surface is $+\cdot0567\theta^2$, the depth of the surface of best definition is now $-\cdot0164\theta^2$ (*i.e.* concave towards the incident light) and a flat photographic plate could lie between the focal lines.

2.3. *Design of Anastigmatic Monochromat Lens.*—To construct the see-saw diagram for a system of lenses we need an expression for the strength of the anastigmatising plate required by a spherical refracting surface. Consider a sphere of radius ρ, refractive index N', immersed in an ocean of index N. Let an object in N-space distant u inside the pole produce an image in N'-space distant v inside the pole. Then the strength of the anastigmatising plate required at the centre of curvature in N-space is

Retardation at zonal radius r

$$= \frac{r^4}{8\rho^3}(N'-N)\left(\frac{N}{N'}\right)^2\left(\frac{u}{u-\rho}\right)^2\left\{\frac{Nu-(N+N')\rho}{Nu}\right\}$$

$$= \frac{r^4}{8\rho^3}(N'-N)\left(\frac{N}{N'}\right)^2\left(\frac{v}{v-\rho}\right)^2\left\{\frac{N'v-(N+N')\rho}{N'v}\right\}$$

while if the plate is placed in N-space, the factor $\left(\frac{N}{N'}\right)^2$ must be replaced by $\left(\frac{N'}{N}\right)^2$.

As a simple example consider an equiconvex lens of thickness equal to the paraxial radius of curvature, ρ of either surface. Let it be used to image parallel light, and set $N=1$, $N'=1\cdot5$, and $\rho=1$. The first surface has $u=-\infty$, $v=\rho N'/(N'-N)=3$, and requires a plate of retardation

$$(N'-N)\frac{r^4}{8\rho^3}\left(\frac{3}{2}\right)^2 \cdot 1 \cdot 1,$$

placed in N'-space at the centre of curvature—that is, on the second surface.

The second surface has $v=-2$ and requires a plate of retardation

$$(N'-N)\frac{r^4}{8\rho^3}\left(\frac{3}{2}\right)^2\left(\frac{-2}{-3}\right)^2\left\{\frac{-6/2-5/2}{-6/2}\right\} = \frac{r^4}{8\rho^3}(N'-N)\cdot\frac{11}{6},$$

placed in N'-space—on the first surface. Thus the second surface should be aspherised with 9/4 times "parabolic correction" and the first surface with 11/6 times "parabolic

correction" in order to produce an anastigmat. Since this arrangement produces anastigmatism by providing the individual anastigmatising plates—annulling the errors *in situ*, so to say—we have no need in this case to image any of the plates into star-space, though it is generally necessary to do so. Notice that the figuring on the second surface is easily null tested, for it is the Cartesian hyperboloid which will produce a stigmatic image in air from parallel light in glass. This lens has $F=6/5$, Petzval radius $3/2=5/4\ F$.

Another simple method of achieving anastigmatism is to place the second surface at the centre of curvature of the first and to figure it with strength $\left(\frac{N'}{N}\right)^2$ times $\frac{r^4}{8\rho_1{}^3}$ (ρ_1 being the radius of curvature of the *first* surface).

This anastigmatises the first surface: we then choose the radius of curvature of the second surface so that its anastigmatising plate has zero strength—*i.e.* we set

$$N'v=(N+N')\rho_2.$$

With $\rho_1=1$, $N'=1\frac{1}{2}$, $N=1$ we have $v=-2$ and $\rho_2=6/5$ concave. This lens has $F=9/2$, Petzval radius $15=10/3\ F$. These two designs, though probably of no great practical value, should form, with what has gone before, a sufficient illustration of the application of the method to lens systems.

3. *The Higher Order Aberrations.*—The validity of the preceding discussion holds only for the extended paraxial region; outside this region the see-saw diagram will give at best an approximation, and, it may be, only a very rough approximation, to the performance of the system. It might be possible to develop methods of successive approximation, taking into account sixth power terms in the strengths of the plates; curvatures of the plates themselves; curvatures produced by the process of locally Gaussian imaging the plates into star-space, and perhaps even the effect of plates on other plates which are imaged through them.

I do not believe, however, that this line of attack would be a happy one, nor do I believe it exemplifies the most profitable attitude of mind with which to assess the value of the see-saw diagram. The value of the see-saw diagram—such as it is—does not depend on the possibility of its extension to higher-order aberrations, nor yet on its speed in computing the extended paraxial properties of a system. The measure of its value is the degree of comprehension which it affords us, and the speed with which that comprehension is attained, of the properties of a class or range of systems, and of the way in which those properties vary as we proceed along the range. The method should be tried, in fact, with a view to exploiting its suggestiveness and not its brevity. There are, broadly speaking, two ways in which this can be done: one can compute a particular system numerically, and examine which of the weights or distances play a predominant part in the behaviour of the see-saw, making it easy or difficult as the case may be, to balance. This process led to § 2. Alternatively, one can observe that the properties of the see-saw for this class of system are peculiarly amenable to algebraic treatment. Dr. E. H. Linfoot has carried out a complete discussion of the class of systems consisting of two mirrors and one aspheric plate, along these lines.

Let us accept, then, that the proper use of the see-saw lies essentially in extended paraxial discussions—that is, to suggest limited ranges of design worthy of more exact investigation by computational methods. The problem involved is always the same: how big are the higher-order aberrations (6th, 8th, etc. powers); to what fraction can we reduce the total aberration by "backing off" against the 6th and 8th powers a judicious mixture of 2nd and 4th power aberrations, and what is the recipe for the mixture in question? This can be studied in two ways. I am personally attracted by the experimental method, in which the off-axis errors of the system are studied during figuring, and rough estimates made of that part of the error which can be regarded as simple

spherical aberration, simple coma, and simple astigmatism. Consideration of the see-saw diagram then provides a convenient method of calculating what figurings or changes of working distances will be required to equilibrate these errors. Off-axis observation may also show that the residual error can be reduced by a sixth-order figuring on some particular surface. I would hardly advocate this method in purely spherical surface work, but it may become reasonable when aspheric surfaces are employed.

Others will prefer to carry computation to its limit, so that in principle no experimental work remains. Let us accept with gratitude any design data offered by these more able analysts: life is short, but technology is long.

Summary

The optical see-saw diagram provides a method of describing the imaging properties of a system of centred aspheric surfaces in the extended paraxial region. The description is given in terms of the anastigmatising plates ("Schmidt plates"), which each surface would need, if used alone, to remove its third-order errors, and of the figurings of the individual surfaces. The method is applied to the design of a "Baker" reflecting anastigmat; to an approximately anastigmatic system, and to two anastigmatic "singlet" lenses.

SOME RECENT CHANGES IN THE SPECTRUM OF γ CASSIOPEIÆ

E. M. Peachey, B.Sc.

(Communicated by the Director, University of London Observatory)

(Received 1942 May 7)

A number of spectra of γ Cassiopeiæ have been taken from time to time at the University Observatory since the installation, in 1935, of the 4-prism spectrograph by M. Tremblot, and for some time it has been suspected that appreciable changes in the star's spectrum occurred at quite short intervals, possibly as short as one or two days. It was hoped that it would still be possible to obtain occasional spectra of this star after the closing of the observatory in August 1939, in order that no important changes might go unnoticed.

Plates taken on 1940 July 8 and 11 showed a definite change in the appearance of $H\gamma$ during this three-day interval; accordingly, it was decided to make observations as frequently as possible. Between July 8 and November 22, 35 plates were obtained which have now been measured on the Hilger machine.* The measured wave-lengths of some of the principal lines, corrected for a radial velocity of $-6{\cdot}8$ km./sec., are given in the table, together with their probable errors.

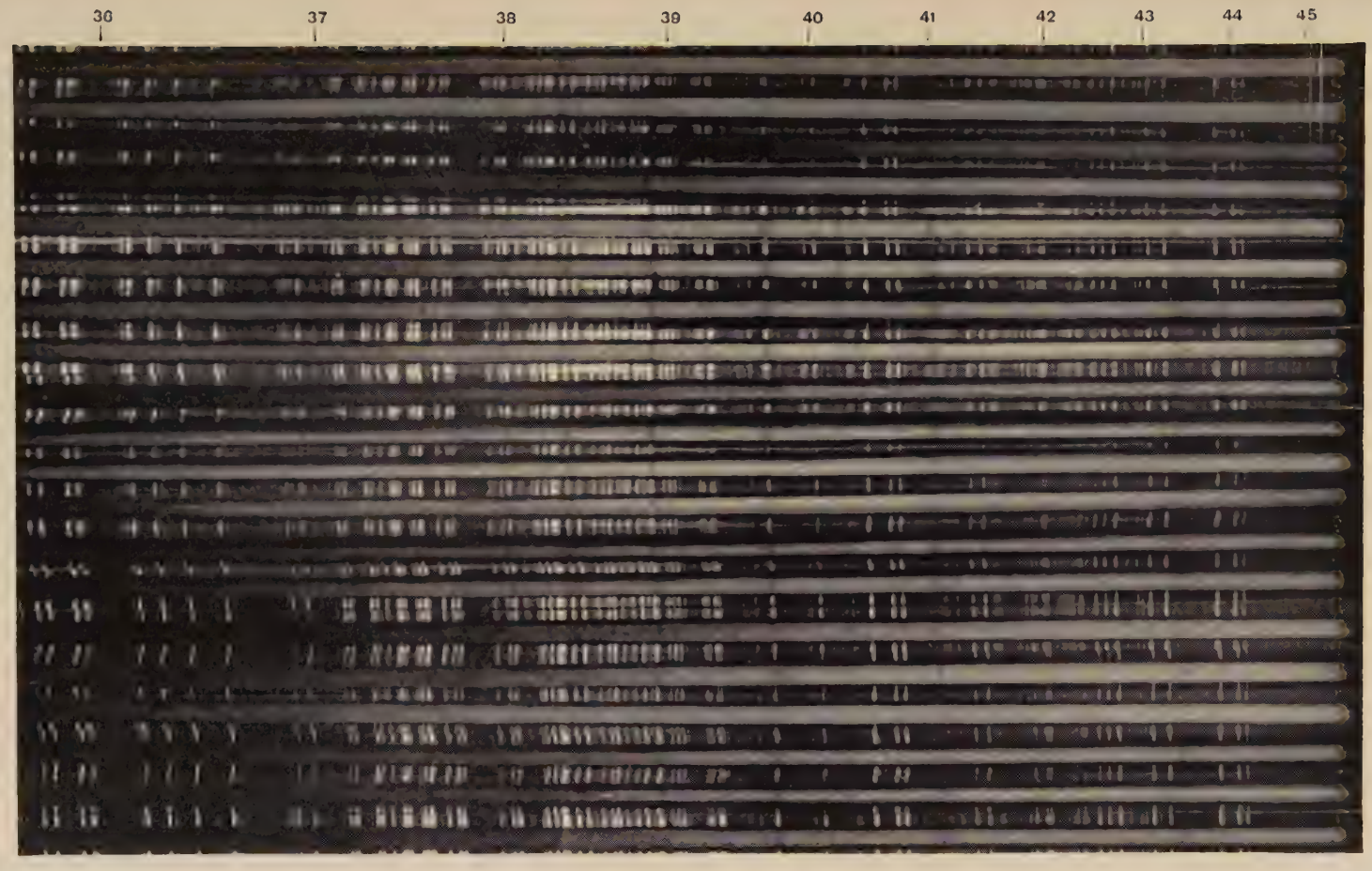
Line	Number of Plates	Mean Wave-length corrected for Radial Velocity λ_0	Probable Error	Tabulated Wave-length λ_τ	$\lambda_0 - \lambda_\tau$
*He*I 4471	36	4471·26	±·068	4471·48	−·22
$H\gamma$	32	4340·45	±·033	4340·48	−·03
$H\delta$	36	4101·86	±·045	4101·75	+·11
*He*I 4026	36	4026·30	±·059	4026·19	+·11
$H\epsilon$	36	3970·17	±·044	3970·08	+·09
*He*I 3965	11	3964·69	±·050	3964·73	−·04
$H\zeta$ + *He*I 3889	36	3888·61	±·025	$H\zeta$ 3889·05 *He*I 3888·65	...

During the period covered by these plates, the hydrogen lines of the Balmer series varied from very sharp, strong absorptions with faint emission borders to broad, fainter, diffuse absorption lines. The lines of Helium I behaved similarly, but to a much smaller extent. Sharp absorption lines had been seen to appear at the end of 1939, following the emission-line stage.†

Eighteen spectra, selected as being typical, are reproduced in the accompanying plate. Two additional spectra, Nos. 1 and 2, taken during the summer of 1939 by Mr. Rahman, are included to show the transition from the emission-line stage to the sharp absorption stage. Much detail clearly visible on the original negatives is necessarily lost in reproduction, but nevertheless it is hoped that some of the changes described may be seen—in particular the general broadening of the lines and their subsequent narrowing on Nos. 18 and 19, before their return to normal on November 22. Variations in the strength and

* Lent to the Director by the Royal Society.

† Baldwin, *Ap. J.*, **92**, 82, 1940.



γ Cassiopeiæ.

1 1939 July 13.
2. ,, Aug. 22.
3. 1940 June 17.
4. ,, July 11.
5. ,, Aug. 31.
6. 1940 Sept. 1.
7. ,, ,, 3.
8. ,, ,, 4.
9. ,, ,, 9.
10. ,, ,, 13.
11. 1940 Sept. 17
12. ,, ,, 21.
13. ,, ,, 30.
14. ,, Oct. 2.
15. ,, ,, 11.
16. 1940 Oct 14.
17. ,, ,, 25.
18. ,, Nov. 19.
19. ,, ,, 20.
20. ,, ,, 22.

In examining the spectra, due allowance has been made for variations in plate density.

In the general description, the Helium I lines 4388 and 4144 have not been mentioned. These were very faint, and were not always visible; in general, they appeared as fairly broad absorption lines, being rather stronger than usual on September 24 and October 19. *He*I 4009, which belongs to the same series as these two lines, was also observed occasionally as a very faint absorption.

The hydrogen lines *H*9, *H*10, *H*11, *H*12 and *H*13 were measured on the denser plates: they appeared as very broad, diffuse, faint absorptions, sometimes with narrow absorption cores. *He*I 3820 (which belongs to the same series as *He*I 4471 and 4026) also appeared as a broad, diffuse, faint absorption line.

It seems desirable that more plates, preferably obtained with an instrument of higher dispersion, should be taken in any subsequent period of the star's history during which rapid changes are found to occur.

In conclusion, I am grateful to the Director for the facilities for pursuing the above investigation, for helpful suggestions and criticisms during the course of the work, and for help in taking some of the plates.

Summary

During the summer and autumn of 1940, while the spectrum of γ Cassiopeiæ was devoid of conspicuous bright lines, a number of spectra of this star were obtained showing changes of a rapid and irregular nature in the appearance of some of the lines of hydrogen and neutral helium. These changes are described and illustrated by a plate showing some of the spectra. The mean wave-lengths of seven of the principal lines are given with their probable errors.

University of London Observatory:
1942 *May* 5.

MEAN AREAS AND HELIOGRAPHIC LATITUDES OF SUNSPOTS IN THE YEAR 1938

Royal Observatory, Greenwich

(Communicated by the Astronomer Royal)

(Received 1942 April 14)

The following results are in continuation of those given in *M.N.*, **100**, 495, 1940, and are derived from the measurement at Greenwich of photographs taken at the Royal Observatories of Greenwich and the Cape and at the Kodaikanal Observatory, India.

Table I gives the mean daily areas of umbræ, whole spots and faculæ for each synodic rotation of the Sun included in the year 1938; the means for the year as a whole are included in Table II, which summarizes the yearly values from 1933–38.

TABLE I

No. of Rotation	Rotation Commenced U.T.	Days Photographed	Mean Daily Areas: Projected *			Mean Daily Areas: Corrected for Foreshortening †		
			Umbræ	Whole Spots	Faculæ	Umbræ	Whole Spots	Faculæ
1127	1937 Dec. 11·86	27	359	2175	2250	261	1598	2716
1128	1938 Jan. 8·19	28	560	3114	2174	420	2370	2609
1129	Feb. 4·53	27	406	2386	2777	291	1717	3326
1130	Mar. 3·86	27	390	2095	3006	286	1530	3695
1131	Mar. 31·17	28	558	2964	2845	439	2356	3469
1132	Apr. 27·44	27	655	3309	2961	475	2392	3668
1133	May 24·66	27	399	1926	2579	297	1451	3227
1134	June 20·86	27	801	4234	2548	559	2944	3207
1135	July 18·06	27	549	3049	3067	444	2548	3975
1136	Aug. 14·28	28	346	1750	2885	254	1260	3658
1137	Sept. 10·53	27	362	1983	2341	279	1613	3000
1138	Oct. 7·81	27	443	2653	2230	308	1813	2747
1139	Nov. 4·11	28	647	3572	2122	458	2573	2624
1140	Dec. 1·42	27	481	2574	2165	352	1939	2675

TABLE II

Year	No. of Days: Photographed	No. of Days: Without Spots	Mean Daily Areas: Projected *			Mean Daily Areas: Corrected for Foreshortening †		
			Umbræ	Whole Spots	Faculæ	Umbræ	Whole Spots	Faculæ
1933	365	234	24	120	225	18	88	267
1934	365	145	27	142	297	22	119	354
1935	365	21	152	810	910	117	624	1100
1936	366	0	288	1529	2106	217	1141	2545
1937	365	0	509	2795	2856	375	2074	3505
1938	365	0	501	2706	2578	371	2019	3205

* Expressed in millionths of the Sun's disk.

† Expressed in millionths of the Sun's hemisphere.

Table III gives for each rotation in the year 1938 the mean daily area of the whole spots (corrected for foreshortening) and the mean heliographic latitude of the spotted areas for both northern and southern hemispheres.

TABLE III

No. of Rotation	Rotation Commenced U.T.	Spots North of the Equator		Spots South of the Equator		Mean Latitude of Entire Spotted Area	Mean Distance from Equator of all Spots
		Mean Daily Area	Mean Heliographic Latitude	Mean Daily Area	Mean Heliographic Latitude		
			°		°	°	°
1127	1937 Dec. 11·86	990	18·03	608	17·83	+ 4·39	17·95
1128	1938 Jan. 8·19	2187	17·15	183	13·46	+14·78	16·86
1129	Feb. 4·53	486	13·55	1232	17·82	− 8·95	16·62
1130	Mar. 3·86	405	20·26	1125	15·16	− 5·78	16·52
1131	Mar. 31·17	617	24·05	1739	15·40	− 5·06	17·66
1132	Apr. 27·44	617	18·64	1776	16·44	− 7·39	17·01
1133	May 24·66	1065	12·54	386	12·18	+ 5·96	12·45
1134	June 20·86	953	12·02	1992	13·35	− 5·14	12·92
1135	July 18·06	698	10·63	1851	14·98	− 7·97	13·79
1136	Aug. 14·28	727	11·82	533	17·92	− 0·76	14·40
1137	Sept. 10·53	789	15·13	824	11·17	+ 1·70	13·11
1138	Oct. 7·81	1453	17·37	360	12·42	+11·46	16·39
1139	Nov. 4·11	1078	14·72	1495	10·00	+ 0·35	11·98
1140	Dec. 1·42	650	14·92	1288	11·57	− 2·69	12·70

The mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots are also tabulated. The mean values for the year 1938 are included in Table IV.

TABLE IV

Year	Days Photographed	Spots North of the Equator		Spots South of the Equator		Mean Latitude of Entire Spotted Area	Mean Distance from Equator of all Spots
		Mean Daily Area	Mean Heliographic Latitude	Mean Daily Area	Mean Heliographic Latitude		
			°		°	°	°
1933	365	86	10·60	2	8·35	+10·23	10·56
1934	365	44	16·45	74	28·09	−11·49	23·75
1934	Old Cycle	19	4·10	1	4·37	+ 3·79	4·11
	New Cycle	26	25·44	74	28·31	−14·46	27·57
1935	365	205	22·42	419	23·72	− 8·57	23·30
1936	366	463	19·36	678	21·02	− 4·63	20·35
1937	365	1317	17·63	757	15·96	+ 5·36	17·02
1938	365	899	15·55	11·20	14·17	− 0·94	14·79

Tables II and IV are in continuation of similar tables given in *Monthly Notices*: for the years 1874 to 1888—**49**, 381–382; 1889 to 1902—**63**, 465–466; 1901 to 1914—**76**, 402–403; 1913 to 1924—**85**, 1007–1008; 1923 to 1933—**94**, 870–871.

The rotations in Tables I and III are numbered in continuation of Carrington's series.

The chief features of the record of sunspots and faculæ for the year 1938 are as follows:—

1. For the mean daily area of sunspots a very high value was recorded only slightly less than that of 1937, with the highest peak since the Greenwich record was begun in 1874. These two years, 1937 and 1938, constitute a notable sunspot maximum comparable with the outstanding peak of 1870 (Kew photographs) and those of 1837, 1788 and 1778 (Wolf's numbers). The mean daily value of faculæ (3205) shows a small proportional decrease on that for 1937.

2. On no day of the year were either sunspots or faculæ absent from the Sun's disk.

3. In 1937 the northern hemisphere was markedly the more active. During 1938 the greater activity was in the southern, but the disparity between the two hemispheres was less marked, the ratio being $1 : 1\frac{1}{4}$.

4. The number of spot groups lasting for two days or longer was 469, of which 252 were north of the Sun's equator and 217 south of the equator. The excess of 221 units in mean daily *area* of the southern hemisphere over the northern is chiefly due to the greater number of medium-sized to large groups (not the very largest) in the southern hemisphere.

During 1938 there were 30 spot-groups $\geqslant$ 500 millionths of the Sun's hemisphere in mean area. Of these there were nine of mean area $>$ 1000 millionths. One of these great groups (C.M.P. Jan. 18·4 U.T.) had a mean area of 2955 millionths, which places it as the second largest group ever recorded at Greenwich, both according to the maximum area reached and the mean area during transit of the disk.

5. The mean latitude of all spots weighted according to area is 14°·8, and thus continues the equatorward progression of the mean spot belt in the two hemispheres (north and south) during the march of the 11-year cycle. Latitudes for individual spots ranged from 0°·7 to 45°·6. These refer to two one-day spots. For spots of duration two days and over the range is 1°·8 to 42°·1.

Royal Observatory,
Greenwich:
1942 *March* 27.

OBSERVATIONS OF OCCULTATIONS AT SYDNEY OBSERVATORY DURING 1941

(Communicated by the Acting Government Astronomer)

(Received 1942 March 18)

The following observations of occultations were made by H. W. Wood (H. W.) and E. E. Adderley (E. A.) and reduced in duplicate by H. W. Wood. The observations were made with a 6-inch equatorial telescope, using a tapping key to record the times on a chronograph, no correction for personal effect being allowed in reading off. The reductions were made by Comrie's method (Supplement to the *Nautical Almanac*, 1938), no adjustment being made to the observed time to allow for error in the Moon's tabular mean longitude.

The observational material is given in Table I, and the results of the reductions in Table II. In each case the phase observed was disappearance at the dark limb. The lunation number and occultation number were taken from the *Nautical Almanac*.

TABLE I

No.	Date	Star	Mag.	U.T.	Observer
				h m s	
384	May 6	37 Sextantis	6·3	11 34 54·5	E. A.
521	July 2	38 Virginis	6·2	9 59 01·4	H. W.
544	July 5	B.D. −14° 4266	6·4	9 23 33·2	H. W.
600	July 29	B.D. −1° 2699	7·0	8 11 22·5	E. A.
627	Aug. 2	ϕ Ophiuchi	4·4	12 37 05·0	H. W.
645	Aug. 4	B.D. −17° 5203	6·0	10 28 42·4	H. W.
705	Aug. 28	B.D. −12° 4214	7·5	8 40 01·7	E. A.
707	Aug. 28	B.D. −12° 4227	7·0	11 51 29·6	E. A.
803	Sept. 23	B.D. −7° 3728	6·5	8 28 43·9	E. A.
805	Sept. 24	ξ^1 Libræ	5·8	9 02 51·1	E. A.
816	Sept. 26	B.D. −16° 4371	6·5	10 24 55·0	H. W.
844	Sept. 29	61 Sagittarii	5·0	13 55 31·3	H. W.
938	Oct. 26	54 Sagittarii	5·4	12 12 06·6	H. W.
939	Oct. 26	55 Sagittarii	5·1	13 04 44·3	H. W.
952	Oct. 28	B.D. −12° 5998	6·8	10 36 59·4	E. A.
953	Oct. 28	B.D. −12° 6005	6·5	11 02 10·6	E. A.
1032	Nov. 22	B.D. −17° 5535	7·4	9 47 02·0	H. W.

TABLE II

No.	Lunation	p	q	p^2	pq	q^2	$\Delta\sigma$	$p\Delta\sigma$	$q\Delta\sigma$	Coefficient of $\Delta\alpha$	Coefficient of $\Delta\delta$
							″	″	″		
384	227	+ 99	+17	97	+17	3	+0·6	+0·6	+0·1	+14·8	−0·13
521	229	+ 84	+54	71	+45	29	+1·3	+1·1	+0·7	+14·5	+0·25
544	229	+ 72	−69	52	−50	48	+2·4	+1·7	−1·7	+ 8·5	−0·82
600	230	+ 84	−54	71	−45	29	+1·0	+0·8	−0·5	+ 9·6	−0·77
627	230	+ 99	−13	98	−13	2	+2·1	+2·1	−0·3	+14·0	−0·26
645	230	+ 74	+67	55	+50	45	−1·2	−0·9	−0·8	+10·4	+0·68
705	231	+100	− 8	99	− 8	1	+1·0	+1·0	−0·1	+13·9	−0·30
707	231	+ 37	+93	14	+34	86	−1·0	−0·4	−0·9	+ 8·2	+0·83
803	232	+ 92	+38	85	+35	15	−0·2	−0·2	−0·1	+14·8	+0·11
805	232	+ 84	+55	70	+46	30	−0·8	−0·7	−0·4	+13·8	+0·34
816	232	+ 43	+90	18	+39	82	−1·1	−0·5	−1·0	+ 7·4	+0·86
844	232	+ 54	+84	29	+45	71	+0·2	+0·1	+0·2	+ 5·9	+0·91
938	233	+ 94	+35	88	+33	12	−0·4	−0·4	−0·1	+12·7	+0·46
939	233	+ 82	+57	68	+47	32	−1·6	−1·3	−0·9	+10·7	+0·67
952	233	+ 97	−25	94	−24	6	+1·4	+1·4	−0·4	+14·7	−0·01
953	233	+ 99	−12	99	−12	1	+0·6	+0·6	−0·1	+14·6	+0·12
1032	234	+ 98	−22	95	−22	5	+1·3	+1·3	−0·3	+14·2	−0·13

OCCULTATIONS OF STARS BY THE MOON OBSERVED AT THE NIZAMIAH OBSERVATORY, HYDERABAD, DURING THE YEAR 1941

(Communicated by the Director)

(Received 1942 June 2)

TABLE I

No.	Star	Mag.	Phase	Date 1941	U.T. h m s
1	B.D. +1° 28	7·3	D	Jan. 4	16 19 39·9
2	31 Arietis	5·7	D	7	17 17 26·2
3	B.D. −18° 4516	6·3	R	23	23 52 12·8
4	B.D. −12° 4134	6·4	R	Mar. 16	20 00 30·1
5	η Libræ	5·6	R	17	18 11 19·0
6	75 Tauri	5·3	R	July 19	23 06 45·7
7	B.D. −16° 5481	8·7	D	Sept. 29	13 29 43·1
8	B.D. −17° 5832	7·4	D	29	13 42 25·3
9	B.D. −14° 5908	6·6	D	30	17 46 17·1
10	B.D. −18° 5063	9·0	D	Oct. 25	14 05 03·8
11	B.D. −18° 5073	8·3	D	25	14 13 11·8
12	B.D. −18° 5079	6·5	D	25	14 34 57·3
13	B.D. −18° 5080	8·5	D	25	14 39 18·0
14	B.D. −18° 5078	9·0	D	25	14 45 26·3
15	B.D. −18° 5086	8·7	D	25	15 02 46·5
16	B.D. −18° 5091	8·8	D	25	15 13 40·1
17	B.D. −17° 5743	9·0	D	26	14 05 29·8
18	B.D. −17° 5746	7·0	D	26	14 27 10·1
19	B.D. −17° 5747	9·5	D	26	14 32 35·1
20	B.D. − 4° 5868	6·6	D	30	18 03 08·3
21	B.D. − 1° 4525	6·3	D	31	15 21 04·2
22	B.D. −13° 5903	9·3	D	Nov. 24	13 30 44·1
23	B.D. −13° 5904	6·7	D	24	13 40 31·2
24	B.D. −13° 5910	8·6	D	24	14 08 33·7
25	B.D. − 9° 5948	6·1	D	25	17 03 20·8
26	B.D. +12° 436	5·8	D	Dec. 1	20 48 29·8
27	5 Cancri	5·9	R	7	17 10 03·4
28	89 Leonis	5·8	R	11	23 26 47·0
29	B.D. −15° 5848	6·0	D	21	14 35 25·7

Two observations (Nos. 1 and 2) were made by M. V. V. Sastri and the rest by M. K. Bappu. The instrument used throughout was the 15-inch visual refractor. The clock corrections were determined from observations with the transit instrument.

The reductions were carried out by M. V. V. Sastri assisted by S. Aravamudhan and V. V. Raghavaiah. No correction has been applied to the observed time in the reduction of occultations.

[TABLE II.

TABLE II

No.	$\sigma' - \sigma$	$\cos(\rho - \chi)$	$\sin(\rho - \chi)$	No.	$\sigma' - \sigma$	$\cos(\rho - \chi)$	$\sin(\rho - \chi)$
	″				″		
1	+1·4	+0·88	−0·47	16	+1·4	+1·00	−0·03
2	+0·8	+0·89	+0·46	17	+0·9	+0·99	+0·13
3	−1·0	−0·75	+0·66	18	−2·0	+0·93	+0·37
4	−1·3	−0·96	+0·28	19	−2·1	+1·00	−0·01
5	+0·2	−0·92	−0·39	20	−1·2	+0·78	+0·62
6	+0·8	−0·22	−0·98	21	0·0	+0·93	+0·38
7	−0·7	+0·87	+0·50	22	+2·9	+0·98	−0·17
8	+0·9	+0·73	−0·68	23	−0·6	+0·93	+0·37
9	−0·9	+0·48	+0·88	24	+2·3	+0·95	+0·31
10	+0·5	+0·61	−0·79	25	−0·1	+0·94	+0·34
11	+1·8	+0·93	+0·37	26	+0·9	+0·95	+0·32
12	+1·2	+0·91	−0·41	27	−0·2	−0·92	−0·38
13	+2·1	+0·84	−0·54	28	−1·1	−1·00	+0·03
14	+0·5	+0·58	−0·82	29	+3·0	+0·84	−0·55
15	−2·1	+0·79	+0·62				

The mean places for 1941·0 of twelve stars in the above list are given in the following table. For stars Washington 7517, and Tacubaya $-14°\ 21^h\ 12^m$, 114, proper motions have been applied to the co-ordinates in the catalogue in computing the mean places for use in the reductions.

TABLE III

No.	R.A. (1941·0)	Dec. (1941·0)	Remarks
	h m s	° ′ ″	
7	19 57 50·74	−16 43 48·6	Washington 7517. P.M. $\mu_\alpha - 0^s{\cdot}0035$; $\mu_\delta - 0''{\cdot}088$.
10	18 44 5·72	−18 49 8·0	Alger 7894.
11	18 44 37·75	−18 29 22·1	Alger 7902.
13	18 45 21·41	−18 41 56·0	Alger 7909.
14	18 45 15·71	−18 46 0·3	Alger 7906.
15	18 45 57·25	−18 21 1·4	Alger 7911.
16	18 46 35·25	−18 30 16·4	Alger 7921.
17	19 43 19·14	−17 19 33·1	Washington 7434.
18	19 43 45·57	−17 13 27·0	Washington 7439.
19	19 44 1·95	−17 18 43·6	Hyderabad $-17°$ 59792; $-18°$ 59003.
22	21 16 5·66	−13 27 39·2	Tacubaya $-14°\ 21^h\ 12^m$; 96.
24	21 16 48·85	−13 14 13·8	Tacubaya $-14°\ 21^h\ 12^m$; 114. P.M. $\mu_\alpha + 0^s{\cdot}0031$; $\mu_\delta + 0''{\cdot}026$.

Nizamiah Observatory,
Hyderabad (Deccan), India:
1942 *April* 1.